

Intermediation and steering: Competition in prices and commissions*

Tat-How Teh[†]

Julian Wright[‡]

February 27, 2020

Abstract

We explore the implications of steering by an informed profit-maximizing intermediary. The intermediary steers consumers by recommending firms taking into account both the commissions firms offer and the prices they set. Such steering results in higher commissions and consumer prices, so that consumers only benefit from intermediation when their search cost is sufficiently high. Steering reverses the normal relationship between competition and price, with prices increasing in the number of competing firms. We use the framework to study various policies including commission caps (absolute or relative), commission disclosure, promoting information provision, and penalties for inappropriate advice.

JEL classification: D43, D83, L13

Keywords: steering, intermediaries, advice, platforms, commissions

*We thank Simon Anderson, Parimal Bag, Emilio Calvano, Maarten Janssen, Bruno Jullien, Chiu Yu Ko, Gunhaeng Lee, Jingfeng Lu, Michele Polo, Debraj Ray, Patrick Rey, Joel Sobel, Satoru Takahashi, Chengsi Wang, Jidong Zhou, and Junjie Zhou as well as participants in talks at National University of Singapore, Monash University, Collegio Carlo Alberto, EARIE Athens 2018, APIOC Melbourne 2018, ECOP Workshop at the University of Bologna, and Econometric Society Summer School 2019 for helpful comments. An earlier version of this paper was circulated under the title “Steering by information intermediaries”. All errors are ours.

[†]Department of Economics, National University of Singapore, E-mail: tehtathow@u.nus.edu

[‡]Department of Economics, National University of Singapore, E-mail: jwright@nus.edu.sg

1 Introduction

In many markets, consumers are often uninformed about the suitability of the available products before they search, and they rely on advice from information intermediaries. An insurance broker advises on suitable insurance products; a financial broker advises on suitable financial products; a physician advises on suitable treatments and drugs; an online platform ranks sellers for consumers; a search engine ranks ads based on search queries; a specialized retailer advises on which manufacturers' products are more suitable. In these markets, firms (i.e. product suppliers) often offer financial incentives to the intermediaries with the aim of influencing their recommendations. These incentives then induce the intermediaries to direct consumers towards the more profitable products, potentially at the expense of consumers — a phenomenon known as “steering”.

While information intermediaries play an important role in helping consumers find the right product, the possibility of steering has generated considerable interest and concern among academics and policymakers. In health-care, there is a growing concern that medical advice is compromised by kickbacks to physicians from pharmaceutical firms.¹ In insurance and financial advising, the pervasive use of commissions to compensate brokers has led to regulatory responses in some jurisdictions.² In the case of online marketplaces, a key issue is whether in light of their ability to steer consumers to firms that pay higher commissions, whether consumers' best interests are actually well served. The concerns common across these different industries raise several key questions: (i) Do commission payments systematically distort the way intermediaries provide information? (ii) How does steering affect firms' pricing and consumers' search behavior? (iii) Is competition between firms still effective in the face of such steering? (iv) What policies can be used to address distortions arising from commissions and steering?

To answer these questions, we develop a framework of competition between two or more horizontally differentiated price-setting firms, and an information intermediary that receives commissions from firms when consumers purchase from them. The novel feature of our model compared to the previous literature is we look at how firms compete *in prices* as well as commissions when consumers can be steered. Firms' prices matter for an intermediary's recommendations since prices affect the probability consumers will want to purchase from the recommended firm and so the probability that the intermediary receives the respective commission payment. If an intermediary recommends a firm whose price is too high, then after inspection consumers will not purchase the recommended product, and if they stop their search at this point, the intermediary will not obtain any commission. And because commissions act as a marginal cost from the firms' point of view, they also drive up retail prices, thereby leading to an interesting tension for firms as to whether to compete by offering lower prices or higher commissions.

¹Pharmaceutical firms frequently offer physicians benefits in the form of meals, travel, and speaking fees, so as to influence their prescription behaviour, see e.g., Engelberg et al. (2014). In 2007, a settlement reached between the US Department of Justice and the five largest orthopedic device makers required these firms to (i) systematically evaluate their consulting arrangements, (ii) ensure that consulting physicians publicly disclose their financial engagements to their patients, and (iii) publicly disclose the name, location, and amount of money paid to each surgeon or organization (Hockenberry et al., 2011).

²See Oxera (2015) for a discussion of recent regulatory actions with respect to certain insurance and financial products in Australia, The Netherlands, and the United Kingdom, such as banning commissions or requiring that commissions are disclosed. See also Inderst (2015) who surveys the implications of regulating commissions in markets with advice.

More specifically, in our model consumers incur inspection costs to sequentially learn the price and match value of each product (i.e., to search), and they obtain recommendations from the intermediary on the order in which they should inspect the products (i.e. a ranking of products). The intermediary has private information on the consumers' match values and prices of products, but its information on match values is imperfect since it does not observe a component of consumers' match values. This feature means that the intermediary has to take into account the possibility that consumers will not purchase the recommended product if the product doesn't offer high enough surplus. As a result, and in contrast to the existing literature, we provide a model where the incentives of the intermediary to be informative are not exogenously assumed but rather follow from equilibrium considerations.

In our model, the intermediary cannot commit to specific recommendation rules (e.g. an insurance broker meeting a customer to advise on which insurance product to buy). Thus, the intermediary's ranking of products is driven by expected commissions, which depend on both actual commissions and the consumer surplus offered by each product. As a result, firms compete both in commissions and prices. These considerations discipline the intermediary's recommendations and generate a smooth demand function. We initially assume symmetric firms, allowing us to establish the existence of a symmetric pure-strategy informative equilibrium in which consumers rationally inspect only the intermediary's highest ranked product, which is also the highest surplus product in equilibrium.

Under this framework, we investigate the market and welfare implications of steering. We first compare equilibrium outcomes between a hypothetical unbiased intermediary which always provides the best ranking for consumers versus an intermediary which ranks products according to our equilibrium characterization. Our first result shows that when the intermediary steers, it leads to positive commissions and higher prices in equilibrium than the case of no steering. When there is steering, firms use commissions in an attempt to manipulate recommendations so as to (partially) avoid price competition. Firms pass-through these commission expenses as higher prices. In the symmetric equilibrium all firms offer the same high commission, so no firm actually gains an advantage from offering commissions, and indeed they are collectively worse off as a result.

Based on the standard economic intuition, one may think that introducing additional competition among firms can correct the price distortion identified above. However, we show that steering *reverses* the standard (negative) relationship between competition and price. Specifically, when there are more competing firms, each firm has a greater incentive to "cheat" the competition — by offering higher commissions in an attempt to be ranked higher. The greater the number of firms competing, the higher is the equilibrium price to consumers. Thus, our model captures what has been described by insurance industry insiders as "reverse competition," in which competition between insurers has been found to drive commissions up rather than driving prices down (Hunter, 2006).

Steering in our model leads to a reduction in consumer surplus and welfare even though the intermediary provides an unbiased ranking of products in equilibrium. This reflects that the higher prices induced by steering reduce consumption and result in deadweight loss. Given the negative implications of steering identified above, a relevant question is whether the presence of an information intermediary is beneficial. For this, we consider an alternate market without the

intermediary in which consumers search randomly, and therefore suffer from a poorer product match. However, they benefit from a lower final price. Due to these two opposing effects, we show that the presence of an information intermediary lowers consumer surplus and welfare when search costs are low enough, and raises consumer surplus and welfare otherwise.

To curb the negative effects of steering on consumers and welfare, we explore several policy measures. We show mandatory disclosure of commissions, by inducing consumers to inspect beyond the intermediary's highest ranked product, helps lower commission levels and therefore equilibrium prices, which increases consumer surplus and welfare, as well as the total profit of the firms selling via the intermediary. We also show that reducing consumers reliance on intermediaries by enhancing consumer information on their match alternatives and prices (e.g. via neutral review websites providing such information) would have a similar positive effect, as would imposing a penalty for an intermediary caught providing inappropriate advice. Finally, we show a cap on firms' commission levels would be a more direct way to achieve similar effects, but a cap on the price-to-commission ratio (e.g. as in the U.S. 2012 Affordable Care Act) may have the opposite effect.

More generally, our model highlights the importance of policymakers identifying markets with steering, so they can take into account the non-standard ways such markets behave, including that:

- A decrease in the number of firms (e.g. from a consolidation of product suppliers) leads to lower commissions and therefore final prices in contrast to the usual higher prices one would expect in this case.
- While there is double-marginalization as a result of both firms and the intermediary having market power, two-part tariffs would not eliminate this distortion given competition to steer consumers via commissions would continue to inflate prices.
- Indeed, with a large number of competing firms, the outcome approximates the monopoly outcome even though the intermediary has no control over prices;
- Competition between intermediaries may not help the situation.

The rest of the paper proceeds as follows. In Section 1.1 we survey the related literature. We lay out the model in Section 2, which we analyze in Section 3, exploring the implications of steering in Section 4. In Section 5 we analyze the policy measures noted above. In Section 6, we consider three extensions of our framework, to the case in which firms are asymmetric, to the case in which the intermediary sets fees, and to the case in which there are competing intermediaries. Finally, in Section 7 we briefly conclude.

1.1 Related literature

Our work obviously contributes to the burgeoning literature that considers whether intermediaries bias (steer) their advice in favor of firms from which they derive larger revenues. Armstrong and Zhou (2011), de Cornière and Taylor (2019), Hagiu and Jullien (2011), and Inderst and Ottaviani (2012a, 2012b) all consider intermediaries that influence which product or firm consumers buy from and collect commissions from sales.³

³In a slightly different vein, Armstrong and Zhou (2019) take an information design approach and explore how information provision to consumers affects the competition between firms, but they do not consider the role of

In the model of Armstrong and Zhou, (uninformed) consumers are assumed to consider only products/firms recommended by the intermediary even if this may not be rational. This means that the intermediary has no incentive to deliver surplus to consumers and always recommends the product that pays the highest commission, in which case the recommendation carries no informational value. Armstrong and Zhou also find steering leads to higher retail price than a random search benchmark, but because recommendations do not improve product matching, steering is always bad for consumers (relative to the random search benchmark) in their setting.

Like the work of Inderst and Ottaviani, we allow consumers to rationally choose whether to follow the intermediary's recommendation or not, and explicitly model the recommendation as an informative strategic communication. To generate a smooth tradeoff between commissions and recommendations in such a setting, Inderst and Ottaviani assume that intermediaries face an exogenous cost (e.g., a lying or reputation cost) for recommending products that are a worse match. Without such an assumption, they would get that the intermediary always recommends the product that pays highest commission, and the recommendation becomes uninformative in equilibrium. In their setting, the (informative) recommendation is unaffected by price given the intermediary only focuses on product suitability, meaning that in equilibrium firms compete only in commissions and then set the maximum price to extract the entire consumer surplus. In contrast, by taking into account that the intermediary is unsure whether consumers will necessarily buy a product after a recommendation, the intermediary cares about consumer surplus in our setting. This allows us to study the interplay between price competition and commission competition which is missing in the existing literature. It also means that even though we look at similar policy interventions as Inderst and Ottaviani, our results are driven by price effects whereas their results are driven by allocation effects when there are cost asymmetries between firms.

de Cornière and Taylor consider the implications of biased recommendations by a vertically-integrated intermediary in favor of its downstream subsidiary. They show that biased recommendations can be beneficial if downstream firms' strategic instruments are such that an increase in firms' markup also increases the net utility offered to consumers (e.g. pure quality competition), but such biases can be harmful in the reverse case (e.g. pure price competition). The competition for recommendations in our model corresponds to the latter case in their taxonomy, but the theory of harm is different from theirs given that in our setting recommendations are not biased on the equilibrium path. Hagiu and Jullien investigate an intermediary's incentive to garble the consumer search process to prolong their time spent inspecting firms' offerings. Both of these works assume that the extent of "bias" in recommendation is chosen by the intermediary, and then observed by firms and consumers prior to consumers visiting the intermediary. The intermediary trades-off between increasing the per-visit revenue and attracting consumers, and it is maximally biased if the extent of bias is unobservable. The timing in these papers means that firms are unable to influence recommendations, hence the issue of competition for recommendations is moot.

A similar tradeoff between commission payments and the probability of a sale which we focus on has also been considered by Hunold and Muthers (2017). In an example in their paper they consider a retailer picking which of two products to recommend to consumers, in which consumers can also walk away if they don't find the recommended product suitable. However, their example

commission in shaping the intermediary's incentive in information provision.

also relies on the assumption that consumers never buy an unrecommended product, even when they know the unrecommended product is more suitable for them. Moreover, their formulation rules out any interplay between commissions and prices. Finally, the focus of their paper is very different, on how resale price maintenance (RPM) affects market outcomes.

Most of the existing models on steering above assume there are only two products and thus are unable to study the effect of competition on the market outcome. This is because, in markets with steering, the usual approach of comparing the duopoly outcome against the monopoly outcome is not applicable given that steering is not possible when there is only a single product for the intermediary to recommend. To the best of our knowledge the only exception is Armstrong and Zhou (2011), but they rely on numerical simulations to explore how competition affects the market outcome. In contrast, we provide analytical results, showing how competition affects price and commission levels in markets with steering, and so how markets with steering behave differently from standard markets.

A different strand of the recent literature on steering considers design decisions by search engines, and addresses whether these platforms may want to bias their search results (de Cornière and Taylor, 2014; Chen and He, 2011; Eliaz and Spiegler, 2011; White, 2013). In these models, a platform essentially fixes a result quality (or a recommendation) which is observed by firms and consumers before their respective decisions. This means that firms are unable to compete for recommendations, and so any recommendation bias would not arise from financial incentives provided by the firms, but instead arises from the platform's motive to distort firms competition in order to maximize the surplus it can extract through lump-sum fees on firms or through selling ads. In biasing its recommendation, the platform thus faces a tradeoff between fewer buyers using the search engine and higher per-buyer profits generated, which reflects a mechanism that is quite different from our own. A further difference is that in most of these papers except de Cornière and Taylor (2014), result quality is a purely vertical notion while our paper focuses on horizontal differentiation.

2 Model setup

We first lay out our model, before discussing the key assumptions in our setup. Our model has n symmetric horizontally-differentiated firms, a single profit maximizing intermediary M , and a continuum of unit-demand consumers. The intermediary just provides information and collects commissions, but does not set any prices or fees, and we assume its costs are normalized to zero.

Consumers. A consumer l 's taste for each product $i = 1, \dots, n$ is described by the match utility $v_{il} = v_l + \epsilon_{il}$. Here v_l is a consumer-specific component drawn i.i.d across consumers but invariant across products, while ϵ_{il} is a consumer-product match component that is drawn i.i.d across consumers and products. Utility provided by the outside option is normalized to zero. Let F and G be the cumulative distribution function (CDF) for ϵ_{il} and v_l respectively. F has full support over $[\underline{\epsilon}, \bar{\epsilon}]$, while G has full support over $[\underline{v}, \bar{v}]$. We assume $\underline{v} + \bar{\epsilon} \leq 0$ so that for all realizations of ϵ_{il} there are always some consumers who would prefer not to purchase any product. Furthermore, the CDFs F and G are assumed to be twice-continuously differentiable and

increasing, with log-concave density functions denoted as f and g respectively.⁴

Information. Consumers observe nothing (including v_l) initially, and they need to search (i.e., inspect) a product i to learn its price p_i and their match utility v_{il} . Our baseline model allows consumers to either observe only the value of v_{il} , or in addition to observe the exact decomposition of v_{il} when inspecting product i .

M has imperfect information over the product utility of each consumer l in the sense that it observes only the realized components $\epsilon_{1l}, \dots, \epsilon_{nl}$, but not the total match values v_{1l}, \dots, v_{nl} .⁵ Based on its observed information, M provides each consumer a ranking of some or all of the products. This setup allows for the possibility that M provides no information, recommends just a single product (as most of the existing literature has assumed), ranks a subset $k < n$ of the products, or provides a complete ranking of all n products.⁶

Search. After obtaining M 's initial ranking of products, a consumer can choose to follow M 's ranking, start inspecting a randomly selected product, or use the ranking in any other way to determine its order of search. After each inspection, the consumer can either (i) stop searching and purchase one of the inspected products or her outside option or (ii) continue searching if there is any remaining uninspected firm. We can allow M to update its ranking at any point in the consumer's search, and likewise for consumers to make use of any change in M 's ranking to update their preferred order of search, although as we will show this possibility makes no difference to the equilibrium analysis, so we focus on the setup in which M provides its ranking only once.

We assume that each search is costly and that the cost of the first search is not too high so that consumers are always willing to inspect at least one product. The nature of search costs beyond the first search are left unspecified as they do not matter in our baseline model. This reflects the equilibrium property of our model, that consumers will inspect only the product ranked first by the intermediary.

Firms. Firms all have the same constant marginal cost, normalized to zero. In addition to the standard decision of setting a price p_i , each firm i decides the commission τ_i paid to M for every successful purchase. We assume that consumers do not observe these commissions. Given any observed price p_i in the search sequence, consumers hold passive beliefs on the remaining unobserved prices, and on all (unobserved) commissions including firm i 's commission, meaning consumers believe these prices and commissions are equal to their equilibrium levels.⁷

The equilibrium concept we adopt is perfect Bayesian equilibrium (PBE). The timing of the game is summarized as follows: (i) Firms set price p_i and commission τ_i simultaneously; (ii) M observes $(p_i, \tau_i, \epsilon_{il})$ for $i = 1, \dots, n$ and for each consumer l , and provides a ranking of products; (iii) consumers inspect products sequentially. We assume that M cannot commit to specific recommen-

⁴Many widely used distributions such as uniform, normal, exponential, and extreme value have a log-concave density function. See Bagnoli and Bergstrom (2005) for more examples.

⁵As explained in Section 2.1 below, we can allow for a much more general partial information setup that still captures this feature.

⁶In Section A.1 of the Online Appendix we show that M would be worse off if instead it fully revealed all of its information to consumers. We further show in Section A.2 of the Online Appendix that the equilibrium we characterize below in our main proposition (which we call the informative equilibrium with steering) remains an equilibrium in a game with a completely general message space.

⁷Assuming passive beliefs over commissions simplifies our analysis. In Section A.3 of the Online Appendix we consider the alternative specification in which consumers' beliefs are based on the optimality of each firm's commissions given its prices (following McAfee and Schwartz, 1994, and In and Wright, 2018), and show that we can obtain the same results in case F and G are linear.

dition rules that are set before firms decide their prices and commissions — i.e., its recommendation needs to be sequentially rational. In Section A.4 in the Online Appendix, we show our general insights on the implications of steering remain valid if M could somehow ex-ante announce and commit to recommendation rules that are set before firms’ decisions.

2.1 Discussion of modelling assumptions

It is useful to think of our setup as having expert sales agents, each of whom has a local monopoly over a subset of consumers. The agents make recommendations to their consumers on which product they should buy, and sign them up to buy the product from the firm if they agree. Relevant examples here include financial brokers, insurance brokers, and physicians. In these examples, after finding out more about the consumer’s situation, the agent provides the consumer with a recommendation, and receives commissions/kickbacks from firms that also set prices. An alternative example is Google ads. Google ranks firms on how much firms bid to pay them and other criteria (e.g., quality of page, various consumer-specific information) using an algorithm known as Ad Rank. One version of Google ads that fits our setting is fee-per-conversion, where firms pay only when they convert a sale based on a consumer clicking on an advert (which is achieved by tracking the consumer’s activity on the firm’s website).⁸ Finally, the model applies to some supplier-retailer relations in which suppliers control retail prices and the retailer can steer some consumers towards one product or another through recommendations or advice.

The assumption that M knows only consumer-product match components $\epsilon_{1l}, \dots, \epsilon_{nl}$ for each consumer l but not the exact match utility v_{1l}, \dots, v_{nl} captures the fact that in practice intermediaries do not have complete information over each consumer’s preference. As an illustrative example, a consumer might know her average willingness to pay for insurance products, but not know which insurance firm’s product best cater to her need. The insurance broker, on the other hand, is able to identify the best match for each consumer but does not know the consumer’s exact willingness to pay. An equivalent interpretation is that M is unsure whether each product is preferred by consumers over their outside option (i.e., whether $v_l + \epsilon_{il} - p_i > 0$) but knows prices p_i and the consumer-specific “conversion rate” from recommendations to purchases $\Pr(v_l + \epsilon_{il} - p_i > 0)$.⁹

We interpret the consumer-specific utility component v_l in our model as the intensity to which a consumer desires a product (willingness to pay), which is naturally unobserved by M . The heterogeneity and unobservability of v_l implies that M is always unsure whether $v_l + \epsilon_{il} - p_i > 0$ holds for each given i . Without this assumption, M will always recommend the product that pays the highest commission, among the set of products that it knows for sure that are preferred by consumers over their outside option. This creates non-smoothness in the firms’ demand functions and can lead to non-existence of pure-strategy equilibrium in our model.¹⁰ In an earlier version

⁸See <https://support.google.com/adwords/answer/7528254> and the links therein for further details. With some modifications, our framework is also applicable to online-platform settings where the commission is set by the platform rather than the firms. We explore this setting in Section 6.2.

⁹This equivalence is due to the one-to-one relationship between the conversion rate and ϵ_{il} for each given p_i , that is, $\Pr(v_l + \epsilon_{il} - p_i > 0) = 1 - G(\epsilon_{il} - p_i)$. Nonetheless, we stick to our interpretation that M knows $\epsilon_{1l}, \dots, \epsilon_{nl}$ for each consumer but not v_l since it simplifies the exposition throughout.

¹⁰Specifically, each firm enjoys a discrete jump in demand by setting a higher commission than others, leading to a competition dynamic that drives up the commission. However, when the level of commission reaches a sufficiently high level, a firm can profitably deviate by setting its commission close to zero and setting a low price, so as to make sales in case it is the only product that is better than the outside option.

of this paper, we considered an alternative setup in which (i) consumers' private information is instead the draw of their outside option, the value of which is observed by them but not M , and (ii) consumers' first search is free. All of our analysis and results continue to hold in that setup.

More generally, we can allow for consumers and M to each hold partial information regarding the consumer-product match components (before consumers engage in search), which can be combined to determine the true values of $\epsilon_{1l}, \dots, \epsilon_{nl}$. If there is a prior communication stage whereby each consumer can costlessly send a message to M , then there is an equilibrium in which the consumer fully reveals her partial information (while keeping her information on v_l private) so that all our analysis carries over. This reflects that the consumer benefits from greater precision in M 's product recommendations in the equilibrium we characterize. We analyze this prior communication stage in the Appendix, to show this result formally. Therefore, one way to interpret the consumer-intermediary interaction in our model is to imagine that a consumer first communicates with M to reveal the partial information she holds and then M makes a recommendation based on its initial information and the information collected from the consumer.

Following standard discrete choice models, we interpret our model as having a continuum of consumers who have ex-post heterogeneous valuations for each product as captured by the distribution of $\epsilon_{1l}, \dots, \epsilon_{nl}$. Under this interpretation, M tailors its ranking to each different consumer, which fits the setting of expert sales agents. There is, however, an alternative interpretation in which valuations are product-specific but the same across consumers. These valuations are still random with their realizations unobservable by firms and consumers, but observable by M . Under this interpretation, M 's ranking will be the same across consumers, which would fit the setting of non-targeted recommendations.

3 Equilibrium analysis

3.1 Demand derivation

Given firms are symmetric in our setup, it is natural to focus on a symmetric PBE. Let p^* and τ^* be the symmetric equilibrium price and commission levels. To simplify notation, we drop the consumer index l in what follows (the reader should keep in mind that match values and the components of match values are all still specific draws for each consumer).

To characterize a deviating firm i 's demand, we need to characterize M 's optimal recommendation strategy and the corresponding consumers' optimal search behavior for any given set of prices and commissions. To do so it is useful to first note that if a consumer only inspects product i , she will want to purchase it if $v_i - p_i \geq 0$, which from M 's point of view happens with probability $\Pr(v + \epsilon_i - p_i \geq 0) = 1 - G(p_i - \epsilon_i)$. Thus, the expected commission M obtains in this case is $\tau_i(1 - G(p_i - \epsilon_i))$, while the consumer gets a surplus of $v + \epsilon_i - p_i$.

We consider the following proposed symmetric pure-strategy PBE, which we will refer to as the *informative equilibrium with steering* to distinguish it from other possible equilibria.¹¹

¹¹Given the recommendation in our setup is essentially cheap-talk (Crawford and Sobel, 1982), our model will generally have multiple equilibria. For example, there always exists a babbling equilibrium where consumers ignore the ranking and the ranking is pure noise. The equilibrium we characterize is maximally informative in the sense that the intermediary's ranking is in the same order as the ranking of surplus to consumers, and therefore consumers only having to inspect once. Moreover, it is the unique symmetric equilibrium outcome if we focus on equilibria where

- (i) All firms set prices and commissions equal to p^* and τ^* .
- (ii) For each consumer and at any stage in their search process, M ranks all products in order of the expected commission $\tau_i(1 - G(p_i - \epsilon_i))$.
- (iii) Regardless of how many products are ranked, consumers believe that the first-ranked product gives them the highest surplus, and that the surplus of any lower ranked or non-ranked product is (weakly) lower than this.
- (iv) Regardless of how many products are ranked, consumers inspect the highest ranked product (say product i) without searching further, purchasing i if $v + \epsilon_i - p_i \geq 0$, and otherwise purchasing the outside option (in case M makes no recommendation, consumers' purchase and search behavior is optimized as if M is absent).

In what follows, we check that for any realized prices and commissions, M and consumers have no incentive to deviate from the proposed equilibrium strategies, and that the consumers' beliefs are consistent with the equilibrium strategies and Bayes' rule. We first check M 's incentives regarding its ranking. In the proposed equilibrium, consumers only inspect the highest ranked product without searching further. Given this, after observing $\epsilon_1, \dots, \epsilon_n$, the top ranked product i by M satisfies

$$\tau_i(1 - G(p_i - \epsilon_i)) \geq \max_{j \neq i} \{\tau^*(1 - G(p^* - \epsilon_j))\}. \quad (1)$$

In our framework, M cares about the commissions it receives as well as the "conversion rate" from recommendations to purchases. Firm i is more likely to be recommended when it offers higher τ_i or when it is more likely to be bought after being recommended, which depends on the surplus $v + \epsilon_i - p_i$ offered. Given the consumers' beliefs and equilibrium strategies, M cannot do better deviating from this strategy since by construction it cannot achieve more than the maximum expected commission.

Given consumers believe that the first-ranked product gives them the highest surplus, and that the surplus of any lower ranked or non-ranked product is no higher, and given that searches are costly, it is immediate that they will only want to search the highest ranked product, and not search thereafter. In the case that M makes no recommendation, consumers just search optimally based on the particular search technology available.

Finally, we check whether consumers' beliefs are consistent. Consider a consumer that observes that M has ranked product i first. From (1), note that according to the equilibrium strategies of all other players in which prices and commissions are p^* and τ^* respectively, product i is ranked first only if $\epsilon_i - p^* \geq \max_{j \neq i} \{\epsilon_j - p^*\}$, i.e., it provides the consumer with the highest surplus $v_i - p_i$ (given that $v_i = v + \epsilon_i$ and v is constant across products). Thus, the consumer's beliefs in (iv) are correct. Consider now a consumer who has inspected the recommended product and learns v_i and the price p_i . If the consumer observes some off-equilibrium price $p_i \neq p^*$, as noted earlier, we assume that the consumer holds passive beliefs on the unobserved commissions in this case, so she will continue to believe that all firms set their commission equal to τ^* . Consequently, based on M 's equilibrium strategy, the consumer infers that M continues to rank the product with the

the first-ranked product has a strictly higher probability to be inspected by consumers relative to other lower-ranked or unranked products. Section A.5 of the Online Appendix contains further details.

highest surplus first because (1) then implies $v_i - p_i \geq \max_{j \neq i} \{v_j - p^*\}$, and this inference is valid regardless of whether the consumer observes the exact decomposition of v_i or not (recall that our model allows for both possibilities). Therefore, our specification of consumer beliefs in (iv) holds even for non-equilibrium prices.

Given M 's equilibrium strategy, the only product that matters is the product it ranks first for each consumer. For brevity we refer to this as M 's recommended product. Given M 's recommendation and consumers' search behavior characterized above, we know that a product i will be recommended if (1) holds. If we denote $\hat{\epsilon} \equiv \max_{j \neq i} \{\epsilon_j\}$ as the best realization of the consumer-product match component among firm i 's $n - 1$ competitors, and define the cutoff $\bar{x}_i(\epsilon) \equiv -G^{-1}\left(1 - \frac{\tau_i^*}{\tau_i}(1 - G(p^* - \epsilon))\right)$, then (1) can be rewritten as $\epsilon_i - p_i \geq \bar{x}_i(\hat{\epsilon})$. After product i is recommended, it will be purchased if $v + \epsilon_i - p_i \geq 0$ because consumers do not search further. Therefore, firm i 's demand, i.e. the total probability of being recommended and then purchased, is

$$D_i = \Pr(\epsilon_i - p_i \geq -v | \epsilon_i - p_i \geq \bar{x}_i(\epsilon)) \Pr(\epsilon_i - p_i \geq \bar{x}_i(\epsilon)).$$

Given the i.i.d property of match utility draws, $\hat{\epsilon}$ has CDF F^{n-1} , so

$$D_i = \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} [1 - F(\max\{\bar{x}_i(\epsilon) + p_i, p_i - v\})] dF(\epsilon)^{n-1} dG(v). \quad (2)$$

To increase demand, firm i can either (i) decrease its price, which increases both the probability of being recommended and the probability of being purchased conditioned on being recommended; or (ii) increase its commission, which increases the probability of being recommended. As we will show later, in equilibrium each firm adjusts its price and commission such that the effects of these two instruments equalize.

To see how commissions interplay with prices in our model, it is useful to compare our demand structure with the seminal price competition model of Perloff and Salop (1985).¹² Consider the case of a duopoly, so $n = 2$. If we compare the conditions for product selections, the only difference is that $\epsilon_j - p_j$ in the Perloff-Salop model (the net surplus of product j) is replaced by

$$\bar{x}_i(\epsilon_j) = -G^{-1}\left(1 - \frac{\tau_j}{\tau_i}(1 - G(p_j - \epsilon_j))\right), \quad (3)$$

which can be understood as a “commission-adjusted” counterpart of $\epsilon_j - p_j$. If both firms set the same commissions, (3) becomes $\epsilon_j - p_j$. In this case, firm i competes with firm j based on standard price competition, and the demand function coincides with the Perloff-Salop model. If instead firm i offers more commission than its competitor so that $\tau_j/\tau_i < 1$, the “commission adjustment” discounts the net surplus of product j so that (3) becomes smaller than $\epsilon_j - p_j$, which shifts demand towards firm i , giving it a competitive advantage without changing its price.

¹²The original model of Perloff and Salop (1985) did not include any outside option for consumers. However, subsequent studies have extended their original model to allow for an outside option (for a comprehensive survey, see Anderson et al., 1992). Hence, we will still refer to the model with full information as the Perloff-Salop model.

3.2 Equilibrium price and commission

Consider the symmetric equilibrium in which all firms set the equilibrium price p^* and the commission level at τ^* . The profit-maximization problem of a deviating firm i is

$$\max_{p_i, \tau_i} \Pi_i = \max_{p_i, \tau_i} \{(p_i - \tau_i) D_i\}. \quad (4)$$

We derive the first-order conditions corresponding to (4) in the Appendix. After imposing symmetry, the optimality condition for the intermediated price is

$$p^* = \tau^* + \frac{\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} [1 - F(\max\{\epsilon, p^* - v\})] dF(\epsilon)^{n-1} dG(v)}{\int_{\underline{\epsilon}}^{\bar{\epsilon}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} f(\max\{\epsilon, p^* - v\}) dF(\epsilon)^{n-1} dG(v)}, \quad (5)$$

which reflects the standard tradeoff between margin and volume in oligopolies. Hence, firms' equilibrium price equals the sum of the equilibrium commission payment, which acts like a marginal cost for each sale, and the firms' equilibrium markup.

To derive the equilibrium commission, we first note that in equilibrium each firm i chooses a commission that equates demand's positive response to its commission with demand's negative response to its price, i.e.,

$$-\frac{\partial D_i}{\partial p_i} = \frac{\partial D_i}{\partial \tau_i}. \quad (6)$$

To see why this is profit maximizing for firm i , note that if $-\frac{\partial D_i}{\partial p_i} < \frac{\partial D_i}{\partial \tau_i}$, then firm i can increase τ_i and p_i by the same amount, and enjoy the same margin but attract higher demand. By the same logic, when $-\frac{\partial D_i}{\partial p_i} > \frac{\partial D_i}{\partial \tau_i}$, firm i can increase its profit by decreasing τ_i and p_i by the same amount. Hence (6) pins down the equilibrium commission level. After imposing symmetry, (6) can be rearranged as

$$\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} f(\max\{\epsilon, p^* - v\}) dF(\epsilon)^{n-1} dG(v) = \int_{\underline{v}}^{\bar{v}} \int_{p^*-v}^{\bar{\epsilon}} \underbrace{\left[\frac{1}{\tau^*} \frac{1 - G(p^* - \epsilon)}{g(p^* - \epsilon)} \right]}_{=\partial \bar{x}_i / \partial \tau_i} f(\epsilon) dF(\epsilon)^{n-1} dG(v) \quad (7)$$

Expression (7) has an intuitive interpretation. The left-hand side is the density of marginal consumers who are indifferent between i 's product and the next-best offer (which can be the competitors' products or the outside option). Meanwhile the right-hand side is the sensitivity of M 's recommendation to the level of commission ($\frac{\partial \bar{x}_i}{\partial \tau_i}$), multiplied by the density of marginal consumers who are indifferent between i 's product and one of the competitors' products. Intuitively, the commission only shifts the total demand between firms and it does not influence the final purchase decisions of consumers who strictly prefer the outside option over one of the competitors' products.

We assume that the profit function is globally quasi-concave in (p_i, τ_i) , so that the first-order conditions (5) and (7) indeed characterize the equilibrium price and commission. In Section A.6 of the Online Appendix we show that a sufficient but not necessary condition for quasi-concavity is for G to be linear.¹³ Unless otherwise stated, our subsequent results do not rely on this distributional

¹³This assumption is stronger than the log-concavity assumption used in standard discrete choice models, e.g., Caplin and Nalebuff (1991). In our setup firm i 's decision variables enter the argument of $1 - F(\cdot)$ as non-linear terms

assumption provided the equilibrium we characterize exists. The proof of the following proposition is relegated to the appendix (as are all other proofs).

Proposition 1 (*Informative equilibrium with steering*) *The informative equilibrium with steering exists in which:*

1. All firms set $p^* > 0$ and $\tau^* > 0$ given by (5) and (7);
2. M recommends the product with the highest expected commission; and
3. All consumers inspect the recommended product without searching further. Moreover, the solution (p^*, τ^*) to (5) and (7) is unique.

One important implication of Proposition 1 is that introducing an information intermediary into the standard (symmetric) sequential search process can cause consumers to stop searching beyond the recommended product. The search sequence collapses to a single inspection of the recommended product. As discussed earlier, this result is driven by the fact that M , facing uncertain purchase probabilities by consumers after recommendation, partially internalizes consumer surplus. In the symmetric equilibrium where all firms offer the same commission, M 's recommendation is undistorted and it therefore recommends the highest surplus product. Foreseeing this, consumers thus have no incentive to search beyond the recommended product.

Nonetheless, the fact that consumers do not inspect other products does not rule out inter-firm price competition. The reason is that M 's recommendation depends not only on commissions but also prices, so that price competition for consumers manifests itself through competition for M 's recommendation.

The equilibrium in Proposition 1 remains valid even if we allow firms to offer M lump-sum payments along with per-sale commissions (i.e., a two-part tariff). For lump-sum payments to be advantageous, a firm would require that M commits to steer enough consumers to the firm. However, given that M has no commitment power over its recommendation, after accepting the lump-sum payment M can profitably deviate by sometimes recommending other firms to consumers instead.¹⁴ For this reason, firms have no incentive to offer any lump-sum payment to M .

4 Implications of steering

We first explore how steering by M affects market outcomes. A natural benchmark for comparison is to consider an unbiased intermediary that always recommends the highest-surplus product to consumers (no-steering), which may be due to, say, regulations that oblige unbiased advice or

due to the presence of G^{-1} and G , which means that the standard technique developed by Caplin and Nalebuff is not directly applicable. It turns out that log-concavity of f and linearity of G are sufficient for log-concavity of demand function in our environment.

¹⁴Even when M can commit to exclusively recommend the firm offering a lump-sum payment, it may not necessarily be profitable for the firm as the firm would need to compensate M for the total commission that M collects from all other firms. In Section A.7 of the Online Appendix, we show that a lump-sum payment is indeed not profitable under commitment whenever n is sufficiently large.

prohibit commission payments.¹⁵ Alternatively, no-steering may reflect that consumers are well-informed of the product attributes and prices before hand, and hence the intermediary is unable to steer.¹⁶

4.1 Prices and commissions

Consider a hypothetical unbiased intermediary which always ranks products according to the surplus offered to consumers. Given this, it is a strictly dominant strategy for consumers to follow the recommendation and inspect the highest ranked product without searching further. Moreover, since the recommendation is unaffected by commissions, firms have no incentive to pay commission. In this case, our model reduces to the Perloff-Salop model with a consumer outside option. Hence, the unique equilibrium entails zero commission with equilibrium price given by (5) after substituting in that $\tau^* = 0$. If we compare this no-steering equilibrium outcome versus Proposition 1, it is immediate that the commission level is higher when M steers, and we show this leads to a higher price in the following proposition:

Proposition 2 *Compared to the equilibrium without steering, the level of prices and commissions is higher in the informative equilibrium with steering.*

Proposition 2 shows that steering imposes a prisoner's dilemma on competing firms. Under steering, each firm attempts to gain an advantage by competing through commissions, but none of them actually end up with an advantage in the symmetric equilibrium given that all firms offer the same commissions. Indeed, firms are collectively worse-off for two reasons. First, the commission expenses that arise from steering are incompletely passed through to consumers. Second, the resulting higher equilibrium price implies the firms share a smaller total market size.

On the surface, Proposition 2 resembles the conventional double-marginalization problem in which the commission to M is an additional margin that drives up the final product price. In markets with steering, the key distinction is that the margin from commission is voluntarily inflicted by firms themselves, so that a two-part tariff would not necessarily eliminate this double-marginalization. In fact, in any hypothetical equilibrium where no firm offers commission to M , the prisoner's dilemma logic above implies that each firm has a unilateral incentive to deviate by secretly offering commission to influence M 's recommendation.

4.2 Steering welfare away

We now consider the welfare implications of steering. Recall that in the informative equilibrium with steering, M recommends the most suitable product for consumers and consumers only inspect the recommended product. We denote the cost of the first search as s , noting that the equilibrium characterization is independent of s as long as the market is active. Given that consumers search only once in equilibrium, the equilibrium consumer surplus is

$$CS = \int_{\underline{v}}^{\bar{v}} \int_{p^*-v}^{\bar{\epsilon}} [v + \epsilon - p^*] dF(\epsilon)^n dG(v) - s,$$

¹⁵For example, in the United States, FINRA/NASD Conduct Rule 2310 requires brokers and dealers to give suitable advice.

¹⁶We discuss what happens when some consumers are informed and some are uninformed in Section 5

while the profit for firms and M are, respectively,

$$\begin{aligned}\sum \pi_i &= (p^* - \tau^*) \int_{\underline{v}}^{\bar{v}} [1 - F(p^* - v)^n] dG(v) \\ \Pi_M &= \tau^* \int_{\underline{v}}^{\bar{v}} [1 - F(p^* - v)^n] dG(v).\end{aligned}$$

Thus, (total) welfare is

$$W = \int_{\underline{v}}^{\bar{v}} \int_{p^*-v}^{\bar{\epsilon}} [\epsilon + v] dF(\epsilon)^n dG(v) - s.$$

Similarly to Section 4.1, we compare the equilibrium outcome under steering with no-steering. Given the analysis in Section 4.1, it is perhaps not surprising that steering has an adverse impact on consumer surplus and welfare. Formally, we obtain the following result.

Proposition 3 *Compared to the equilibrium without steering, Π_M is higher in the informative equilibrium with steering, while CS , $\sum \pi_i$, and W are lower in the informative equilibrium with steering.*

Some remarks are in order. Even though M recommends the most suitable product in equilibrium, the fact that its recommendation can be influenced via commissions off-equilibrium creates incentive for firms to engage in wasteful competition through commissions. High commissions result in high prices, which implies too many consumers purchase the outside option compared to the efficient solution. To quantify these results, in Section A.8 of the Online Appendix we let F and G be linear with distribution support $[-1, 1]$, showing that steering results in a price increase of at least 36%. The significant increase in prices translates to a corresponding decrease in consumer surplus and welfare of at least 34% and 14% respectively. Nonetheless, it is crucial to note that our results do not necessarily imply a case for abolishing intermediation altogether because the information provided by M still facilitates consumers search. We examine this issue in Section 4.4.

4.3 Competition and market entry under steering

We next show how steering profoundly changes the standard comparative statics of market entry. Define, whenever $\bar{\epsilon}$ is finite, $\tau^m \equiv \arg \max_{\tau} \{\tau(1 - G(\tau - \bar{\epsilon}))\}$ as the commission that maximizes the expected commission that M can collect from a firm with a product of the highest possible value $\bar{\epsilon}$ and which makes no margin (so sets its price equal to its commission). Here, τ^m is equivalent to the profit-maximizing price of a monopolist firm with a product valued at $v + \bar{\epsilon}$ (i.e., with the highest possible level of ϵ) that sells directly to fully informed consumers but where v is still unknown to the monopolist. For this reason, we will refer to it as the “ $\bar{\epsilon}$ -product monopoly price”.

Proposition 4 *Consider the informative equilibrium with steering:*

1. *If $\bar{\epsilon} < \infty$, in the limit as n becomes large, prices and commissions both converge to the $\bar{\epsilon}$ -product monopoly price level τ^m . Formally, $\lim_{n \rightarrow \infty} p^* = \lim_{n \rightarrow \infty} \tau^* = \tau^m$.*

2. *Suppose the inequalities*

$$0 \geq \frac{f'(\epsilon)}{f(\epsilon)} \geq \frac{\gamma'(v)}{\gamma(v)} \quad (8)$$

hold for all $(\epsilon, v) \in [\underline{\epsilon}, \bar{\epsilon}] \times [\underline{v}, \bar{v}]$ such that $v + \epsilon \geq 0$, where $\gamma(v) \equiv \frac{1-G(v)}{g(v)}$ is the inverse hazard rate of G . Then, τ^* and p^* both increase with n .

The first part of Proposition 4 states that as n becomes large, both equilibrium price and equilibrium commission converge towards τ^m . It reflects that, when n is sufficiently large, competition for recommendations becomes so intense that (i) firms compete away all their profit margin so $\lim_{n \rightarrow \infty} p^* = \tau^*$, and (ii) all firms set τ^* that maximize expected commission to M . Point (i) follows from the fact firms' market power, as represented by the markup in (5), vanishes as n becomes large.¹⁷ Point (ii) reflects that the intense competition for recommendations forces each firm to set commissions to maximize its expected commission payment $\tau_i(1 - G(\tau_i - \epsilon_i))$ to M . Crucially, as n becomes large, there will be multiple realized match values from among a firm i 's $n - 1$ competitors for any given consumer that will be arbitrarily close to the distribution's upperbound $\bar{\epsilon}$. This means product i is only recommended to a consumer when its realized ϵ_i is sufficiently close to $\bar{\epsilon}$. Since the price converges to the commission payment as n becomes large, in the limit M will recommend a firm (from among those with match values equal to $\bar{\epsilon}$ for a particular consumer) that offers the highest expected commission $\tau_i(1 - G(\tau_i - \epsilon_i))$. This leads to firms offering τ^m in order to be recommended.

The second part of Proposition 4 extends our analysis to finite n . Condition (8) requires f to be weakly decreasing, but with a gradient that is not too steep relative to the rate of change of the inverse hazard rate of G . For example, if F is linear then $f' = 0$, so that (8) holds for all log-concave $1 - G$, which is implied by our assumption of log-concave g . Similarly, if F is exponential with parameter μ , then $\frac{f'}{f} = -\mu$ so that (8) holds for all log-concave $1 - G$ provided μ is not too large. To see the intuition for the result, recall from (7) that changes in commissions only affect the final purchase decisions of consumers who are indifferent between i 's product and one of its competitor's products. By increasing τ_i , firm i essentially shifts the demand from other firms to itself while keeping the total market coverage fixed. When n is higher, the total market coverage is larger, which amplifies this demand-shifting effect of commissions. All else being equal, this induces firms to raise their commission which then is passed-through into the retail price. At the same time, a higher n reduces firms' markup in the pricing equation. Condition (8) ensures the increase in commission dominates the decrease in firms' markup, and so price monotonically increases in n . Figure 1 below illustrates Proposition 2 and Proposition 4 when F and G are linear with distribution support $[-1, 1]$.¹⁸

The second part of Proposition 4 has implications for competition policy. While the standard economic logic suggests that entry and competition tends to reduce consumer prices, our model predicts that the opposite can happen in markets where firms can compete through commissions. Hence, pro-competitive policies that aim to reduce consumer prices may potentially have the opposite effect on prices in settings where information intermediaries steer consumers.

As for consumer surplus and welfare, there are two opposing effects when n increases. First,

¹⁷This requires that F has unbounded hazard rate $\frac{f}{1-F}$ when evaluated at the distribution upper-bound (see Perloff and Salop, 1985). This condition is satisfied in our model due to our distribution assumptions of full support with finite upper-bound.

¹⁸To illustrate that (8) is sufficient but not necessary for the result, in Section A.8 of the Online Appendix we show a similar figure to Figure 1 in case F and G each take on the standard normal distribution, for which (8) does not hold.

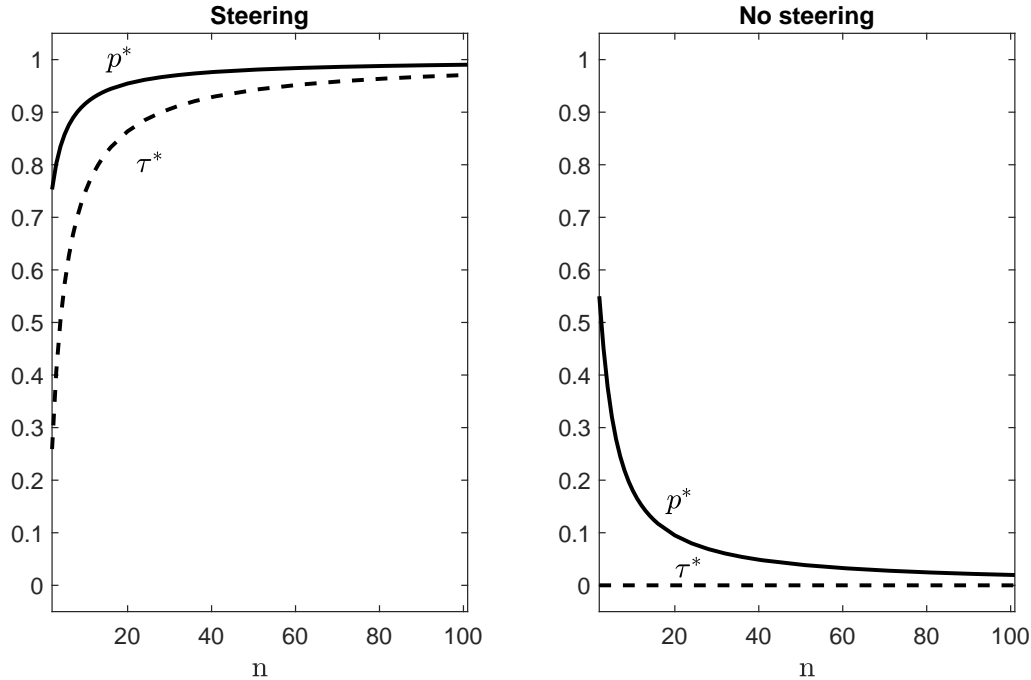


Figure 1: Prices and commissions when M steers versus when it does not.

there is a positive variety effect because consumers can obtain a better match value when there are more firms. Second, equilibrium price tends to increase with n (Proposition 4), which reduces consumer surplus and welfare. Due to these two effects, it is in general ambiguous how firm entry affects consumer surplus and welfare in market with steering.

4.4 Comparison with no intermediation

Given steering lowers consumer surplus and welfare (Proposition 3), a relevant question is whether the existence of M increases or decreases consumer surplus and welfare. To address this question we need to specify the consumers' search technology since without M 's recommendation, consumers will generally want to engage in more than one search. We proceed by adopting the standard framework of Wolinsky (1986) and Anderson and Renault (1999), in which all consumers have a fixed per-search cost $s > 0$ and they search sequentially across firms chosen randomly with perfect recall. To make things consistent with our baseline model, we adapt these models by allowing for the heterogeneous consumer-specific match component, that is, v . The latter feature complicates the analysis, and so for tractability we focus on the limit version with $n \rightarrow \infty$ and assume that consumers fully observe the realization of v after inspecting at least one product.¹⁹ Given the market has infinitely many consumers and firms, without loss of generality, we normalize the number of consumers per firm to one.

The analysis of this model is standard, except that we need to handle the heterogeneity in the consumer-specific match component. We relegate the detailed analysis of this setup to Section A.9 of the Online Appendix. There we show that if s is small, there exists a symmetric equilibrium in which consumers actively search and all firms set the same price. Otherwise, the market is inactive.

¹⁹Recall that in the baseline model, whether consumers observe v after their first inspection or not has no effect on the analysis given that consumers inspect only the recommended product regardless of what they infer about v .

Compared with the equilibrium with intermediation and steering in Proposition 1, we obtain:

Proposition 5 *Suppose consumers face constant search costs s per search, $n \rightarrow \infty$, and F and G are linear with distribution support $[-1, 1]$:*

1. *Focusing on the informative equilibrium with steering when M is present, the equilibrium price is always higher in the presence of M .*
2. *There exist thresholds $\bar{s}_1 > \bar{s}_2 > 0$, such that consumer surplus is lower in the presence of M if and only if $s < \bar{s}_1$, while welfare is lower in the presence of M if and only if $s < \bar{s}_2$.*

Proposition 5 shows that the equilibrium price is higher in the presence of M , which is perhaps not surprising given the limiting result in Proposition 4. Consequently, the presence of M has two effects on consumer surplus. On one hand, consumers benefit from better product matches due to M 's recommendation. On the other hand, they suffer from the higher prices that result from competition in commissions. When search costs are low, consumers can obtain reasonably good product matches by searching sequentially and so M 's recommendation does not improve the product match by much. As a result, consumer surplus is lower with M since the price increase more than offsets the gain in improved product matches. In contrast, when search costs are high, consumers only have limited choices without M 's recommendation, which means the product match improvement from M 's recommendation more than compensates for the higher price. In this case, consumer surplus is higher with M .

For welfare, note that the joint profit of M together with all firms (industry profit henceforth) is maximized when M is present because it results in the \bar{c} -product monopoly price at $p^* = \tau^m$ when $n \rightarrow \infty$. This increase in industry profit due to M explains why the minimum search cost above which M increases welfare is lower than the corresponding minimum search cost above which M increases consumer surplus.

5 Policy implications

In light of our findings in Section 4, in this section we briefly explore six different policy options for dealing with steering.

The first and most drastic option involves banning intermediation altogether, or imposing conditions which would make intermediaries no longer viable. Proposition 5 implies doing so, while always lowering prices, will lower consumer surplus and welfare if consumers' cost of searching directly is sufficiently high.

Another possibility is to ban any commission payments from firms so that the intermediary is forced to generate revenue by charging consumers directly instead (e.g. charging upfront consultation fees).²⁰ Several countries, including Australia, the Netherlands, and the United Kingdom, have introduced such bans on commission payments to advisers for certain types of financial or insurance products (Oxera, 2015). From our model, the switch to consultation fees clearly lowers the

²⁰We are not aware of financial and insurance brokers that charge consumers and at the same time receive commissions from firms. Such "double-dipping" is explicitly banned in some jurisdictions, and opposition to it may explain why intermediaries may not want to charge consumers any fee when they have the option to collect commissions from firms.

equilibrium price and deadweight losses, improving welfare, provided M still recommends the best product when it is indifferent. Meanwhile, given that consumers are homogenous before searching, and the intermediary is a monopolist in our setting, its lump-sum consultation fee would be set so consumers are just willing to come to M rather than search independently (say with random search). Whether the ban benefits consumers depends on how the consultation fee compares to the benefit of lower prices, and in general is ambiguous. If consumers' search cost is sufficiently small, then M 's recommendation does not help to improve the match quality that much relative to what consumers can obtain by searching independently, in which case the consultation fee that M can charge is small so consumers are better off. Formally, our analysis of the case without M in Section 4.4 provides some guidance on this case. For example, if we take consumers' search cost to be arbitrarily close to zero, then Proposition 5 implies consumers are better off with a consultation fee than commissions.²¹

A third possibility would be to increase effort in detecting inappropriate recommendations and/or increase the penalties for an advisor caught recommending a product which is not the best for the consumer so as to make M put more weight on consumer surplus and less weight on commissions in making its recommendations. By reducing the extent of wasteful competition through commissions, this will lower prices and improve consumer surplus, firms' profit and welfare. To formalize this intuition, in Section B.1 of the Online Appendix we consider the case that M , due to its concern for potential penalties on inappropriate advice, cares directly about consumer surplus (on top of its concern for the probability of purchase). We show that when M assigns more weight to consumer surplus, the equilibrium price and commission levels decrease, and consequently consumer surplus, firms' profit, and welfare increase.

A fourth way to achieve lower commissions is to make consumers better informed of their options (product-match and prices) so they are no longer solely reliant on the intermediary for advice. We have in mind websites that provide information to consumers, for instance, with respect to which features of particular products are a good fit for their particular needs.²² This possibility is explored in Section B.2 of the Online Appendix, where we show that equilibrium price and commission levels decrease, and consumer surplus, firms' profit, and welfare increase with the probability of consumers being informed. Intuitively, when a consumer is more informed, she is less reliant on M for recommendations, which makes it less attractive for firms to offer high commissions. This relaxes the competition for M 's recommendation, which then reduces equilibrium commissions and prices. Consequently, the result implies that when consumers become informed, they create a positive spillover for the remaining uninformed consumers. This result suggests that if becoming informed is costly, too few consumers will invest in obtaining the relevant information, which may justify public programs that help consumers determine suitable products.²³

²¹A more complete analysis of consultation fees requires taking into account how the intermediary obtains information. See, in particular, Inderst and Ottaviani (2012b) and Thiel (2019) who show that the shift away from the practice of per-sale commission can decrease consumer surplus by stifling the intermediary's incentive to acquire information or discouraging intermediaries from participating in the market to provide advice.

²²For example, the UK government runs a information website Money Advice Service that provides helpful advice on selecting life insurances and other retail financial products based on different circumstances. In Singapore, the government sponsors a purely informational portal CompareFirst that serves a similar purpose. Likewise, test and review websites for electronic products such as TechRadar and Tom's Guide, among others, allow consumers to explore and compare product attributes and prices before consumers purchase products from specialized retailers. Both TechRadar and Tom's Guide are very explicit in not taking payment for product reviews.

²³For example, the government could consider subsidizing the cost of subscription-based review websites such as

A more direct way to achieve lower commissions is obviously to impose a regulatory cap on commissions. Such a cap on commissions (provided that it is binding) would also improve consumer surplus, firms’ profit, and welfare because it constrains the extent of commission competition without affecting the quality of recommendations, provided M remains viable of course. However, this assumes the regulator imposes a cap on the level of commissions. In practice, regulators may cap commissions as a proportion of the price. For example, the cap on commissions in the U.S. 2012 Affordable Care Act involves the so-called “80-20 rule” that requires insurance companies to spend at least 80% of the premiums paid by policy-holders on health care costs or activities improving quality.²⁴ In terms of our model, the rule can be interpreted as a cap on the commission-to-price ratio, in which the commissions paid to M cannot be more than a fixed proportion η of revenue collected, in this case 20%. Based on our framework, it is easy to show that this “proportional rule”, whenever it is binding, can potentially lead to the unintended consequence of firms increasing their prices in order to compete in commissions while satisfying the proportional rule. This observation is driven by the fact that raising price and commission by the same amount does not violate the rule, and so (6) must hold in equilibrium, whether the rule binds or not. Therefore, the equilibrium commission-price pair $(\bar{\tau}^*, \bar{p}^*)$ satisfies (7) and $\bar{p}^* = \frac{\bar{\tau}^*}{\eta}$, and \bar{p}^* here must be higher than the equilibrium price in the baseline equilibrium (without the proportional rule) given the rule binds.

A final way that we consider for a policymaker to achieve lower commissions is to require mandatory disclosure of commissions. In an attempt to protect consumers of retail financial products, some jurisdictions mandate that brokers and financial advisors disclose to consumers the commissions paid by product providers (Inderst, 2015). To formally analyze the implications of such a disclosure policy, we extend our model by allowing for two types of consumers. A fraction $\lambda \in [0, 1]$ of consumers observe the disclosed commissions and use this in their inference about the surplus of uninspected products. Thus, these consumers may sometimes choose to inspect more than one product. The remaining fraction $1 - \lambda$ of consumers remain uninformed, meaning they do not observe commissions.²⁵ The realization of each consumer’s type is not known to firms and M . Given this model setup, the policy of commission disclosure is equivalent to having $\lambda > 0$, and a higher λ can be interpreted as a more transparent policy. The consumers’ search technology is assumed to be the same as the one described in Section 4.4. For tractability, we assume in addition that the fixed-per search cost s is arbitrarily small, such that consumers continue to search as long as the expected incremental benefit from doing so is strictly positive.²⁶

The full analysis of this setup is relegated to Section B.3 of the Online Appendix. We focus on the following equilibrium (which we call an *informative equilibrium* for short) that is analogous to the equilibrium in the baseline model:

Consumer Reports for consumers.

²⁴See <https://www.healthcare.gov/health-care-law-protections/rate-review/>

²⁵One interpretation of this formulation is that the disclosure policy is only partially effective—only some consumer “see” the relevant information on commissions. An alternative interpretation is that all consumers observe the commission levels, but only a fraction λ of them update their beliefs on M ’s recommendation based on the observed commission, while the remaining $1 - \lambda$ of them are naive (and so do not update their beliefs). Indeed, Chater et al. (2010) provides experimental and survey evidence showing that sometimes disclosed commissions may not become a salient factor in consumers’ decisions, even in face-to-face situations. Lacko and Pappalardo (2004) found evidence that commission disclosure can be confusing to consumers and does not necessarily help them to make better choices.

²⁶The assumption ensures that for all realizations of ϵ there is a strictly positive measure of observant consumers who search more than once. Otherwise, for some realizations of ϵ , M would find it optimal to simply recommend the highest-commission product, which leads to a discontinuity in the firms’ demand function.

- All firms set their prices at the same level and all firms set their commissions at the same level.
- M ranks all products in order of the expected commission $\tau_i(1 - G(p_i - \epsilon_i))$.
- Consumers inspect products in the following way:
 - Unobservant consumers inspect the highest ranked product without searching further. They buy either the outside option or the inspected product, whichever offers the highest surplus.
 - Observant consumers first inspect the highest ranked product. If they infer from the realized match value that v is low enough, they stop after the first search and buy the outside option. Otherwise, they search all products following M 's ranking until they reach the firm that offers the lowest commission, in which case they buy either the outside option or one of the inspected products, whichever offers the highest surplus.

The search strategy of observant consumers can be understood as follows. First, if they infer from the realized match values that v is low (recall that they may or may not observe the decomposition of match values v_i) such that the outside option is relatively more attractive, the consumer has no reason to search further after the first search. Suppose instead the inferred level of v is high. Given that M ranks products according to expected commission and that the search cost is arbitrarily small, a consumer's expected incremental benefit (net of search cost) from an additional search through M 's ranking is strictly positive, as long as she has not reached the lowest commission firm. Once the consumer reaches the lowest commission firm before exhausting M 's ranking, she can infer from M 's ranking strategy that all the lower ranked products must have lower surpluses. Therefore, there is no reason to inspect beyond the lowest commission product given doing so incurs search costs (albeit arbitrarily small search costs). An important implication from this search strategy is that, when commissions are observable, each firm finds it less attractive to offer a higher commission than rival firms. This is because doing so only attracts additional demand from unobservant consumers, given that observant consumers do not necessarily stop searching after inspecting the recommended firm.

It can be shown that the informative equilibrium exists provided that λ is sufficiently small, and its characterization is similar to Proposition 1.²⁷ Based on the equilibrium characterized, we obtain the following result on the effect of commission disclosure.

Proposition 6 *If the informative equilibrium exists, then:*

1. *The equilibrium price and commission levels are lower under mandatory disclosure, while consumer surplus, firms' profit, and welfare are higher under mandatory disclosure.*
2. *The equilibrium price and commission levels decrease with the fraction of observant consumers, while consumer surplus, firms' profit, and welfare increase with the fraction of observant consumers.*

²⁷Due to the co-existence of two consumer types, a firm can potentially deviate from the informative equilibrium by offering no commission and reoptimizing its price given it sells to the informed consumers only. Such a deviation is unprofitable provided λ is sufficiently small.

Proposition 6 is in contrast to the result in Inderst and Ottaviani (2012a), in which they show commission disclosure has no effect on consumer surplus and welfare when firms are symmetric. The reason is that consumer demand is price inelastic in their model, meaning that firms can always extract all consumer surplus in equilibrium. Even though commission disclosure can lead to a lower commission level in equilibrium, those changes are welfare-neutral in their setting given they just represent a profit transfer between firms and the intermediary. In our model with price-elastic consumer demand however, consumer surplus is not fully extracted. Consequently, commission disclosure can lead to a lower final consumer price, as well as higher consumer surplus and welfare due to the resulting reduction in deadweight loss.

6 Extensions

This section explores three extensions of our baseline model. Section 6.1 considers the case of asymmetric firms. Section 6.2 explores the case when M sets the level of commissions. Finally, Section 6.3 explores the case of competing intermediaries. To keep the exposition brief, we focus on presenting the main insights in this section and relegate further details and formal proofs of the propositions to Section C of the Online Appendix.

6.1 Asymmetric firms

In this section, we discuss how the equilibrium characterization in Section 3 can be extended to the case of asymmetric firms. We focus on the case of $n = 2$ firms. Firm $i \in \{1, 2\}$ has constant marginal cost c_i , where $c_2 > c_1 = 0$, so firm 2 is the less efficient firm. Given cost asymmetry, it is natural that firms set asymmetric commissions, meaning consumers may want to search more than once (even if they follow M 's ranking). Whether consumers choose to do so depends on the incremental benefit of a second search relative to the cost of the second search.

We first consider the case where the cost of the second search is sufficiently high so that consumers only search once in equilibrium. We construct the asymmetric (pure-strategy) *informative equilibrium with steering* in what follows. Suppose consumers follow M 's recommendation in equilibrium. Given that consumers only search once, it follows from the baseline model that M recommends firm i if and only if $\tau_i (1 - G(p_i - \epsilon_i)) \geq \tau_j (1 - G(p_j - \epsilon_j))$. Then, we can easily derive the demand and profit functions, and, applying first-order conditions, obtain the best-responding prices and commissions of each firm. Let $(p_1^*, p_2^*, \tau_1^*, \tau_2^*)$ denote the equilibrium prices and commissions. Ex-ante, consumers follow M 's recommendation to inspect firm $i \in \{1, 2\}$ if and only if:²⁸

$$E_{\epsilon_i, \epsilon_j} \left[\epsilon_i - p_i^* - (\epsilon_j - p_j^*) \mid \epsilon_i - p_i^* \geq -G^{-1} \left(1 - \frac{\tau_j^*}{\tau_i^*} (1 - G(p_j^* - \epsilon_j)) \right) \right] \geq 0. \quad (9)$$

This condition holds as long as the difference between τ_i^* and τ_j^* is not too large in equilibrium, and this turns out to be true as long as the cost difference between firms is not too large:

²⁸Alternatively, if consumers are naive and always follow M 's recommendation, as in Armstrong and Zhou (2011), then condition 9 is null, in which case Proposition 7 holds regardless of the cost difference.

Proposition 7 *Suppose F and G are linear²⁹ and the cost of the second search is sufficiently high. If $c_2 - c_1 > 0$ is not too large, there exists an asymmetric informative equilibrium with steering in which:*

1. *Firms set prices $p_1^* < p_2^*$ and commissions $\tau_1^* < \tau_2^*$;*
2. *M recommends the product with the highest expected commission; and*
3. *All consumers inspect the recommended product without searching further.*

Moreover, if $c_2 \rightarrow c_1$ then the equilibrium coincides with Proposition 1.

The equilibrium in Proposition 7 is unique if we focus on equilibria where the recommendation is informative and followed by consumers. A notable feature of this equilibrium is that the less efficient firm sets a higher price and pays a higher commission. Intuitively, if we ignore commissions, the standard cost pass-through logic implies the less efficient firm should charge a higher price. Recall that by paying commission, a firm directly shifts demand from its rival to itself. If the rival has a lot of demand (which happens when the rival is efficient), an increase in commission will have a bigger effect on the firm's demand. This logic implies that firm 2, competing against the more efficient firm 1, has a stronger incentive to pay commission, while the reverse is true for firm 1. The difference in commissions is then passed through into prices, reinforcing the initial effect of the cost difference on the prices. Hence, in equilibrium the less efficient firm indeed charges a higher price.

Next, we consider the case where the cost of the second search is arbitrarily small. In this case, the overall equilibrium in Proposition 7 is no longer sustainable. Suppose consumers expect $\tau_1^* < \tau_2^*$, then they will inspect only firm 1 whenever firm 1 is recommended, but will inspect both firms whenever firm 2 is recommended due to the arbitrarily small search cost. Ignoring the outside option, the search behaviour means firm 2's product is eventually purchased only when it is recommended ($\tau_2(1 - G(p_2 - \epsilon_2)) \geq \tau_1(1 - G(p_1 - \epsilon_1))$) while still providing more surplus than firm 1 ($\epsilon_2 - p_2 \geq \epsilon_1 - p_1$). The second condition implies the first whenever $\tau_2 > \tau_1$, meaning that firm 2 never wants to set its fee above τ_1 as doing so does not generate additional demand. Therefore, $\tau_1^* < \tau_2^*$ cannot be true in any equilibrium.

More generally, the same logic implies if the search cost is arbitrarily small, then regardless of the cost difference there is no (pure-strategy) equilibrium with asymmetric positive commission because the firm being perceived as paying the higher commission can always profitably deviate to a lower commission level. The only pure-strategy equilibrium is an uninformative one, in which both firms compete exclusively through price, while consumers always inspect both products and ignore M 's recommendation. Intuitively, the non-existence of informative equilibria reflects that commission is not an efficient competition instrument when consumers have low search cost and are likely to inspect beyond the recommended firm.

In between these two extreme cases, if the cost of the second search is intermediate, some (but not all) consumers inspect beyond the recommended firm whenever the higher-commission firm is recommended, depending on the realized net surplus of the recommended firm. However,

²⁹In Section C.1 of the Online Appendix we extend the results in Proposition 7 to cases with non-uniform distributions.

pinning down demand functions in such cases is analytically difficult due to two complexities: (i) consumers' reservation value from searching has a mutual dependency with M 's recommendation strategy; and (ii) the product values are correlated due to the recommendation. In Section C.1 of the Online Appendix, we take an alternative approach by assuming there are two groups of consumers: a fraction λ of which have arbitrarily small search cost while the remaining fraction $1 - \lambda$ have sufficiently high search cost and they only search once.³⁰ This allows us to avoid the aforementioned complexities, while at the same time still capturing the feature that some consumers search beyond the recommended firm. We prove that, provided that λ is sufficiently small relative to the cost difference, the informative equilibrium in Proposition 7 can be sustained.

In sum, the analysis in this section suggests that M 's ranking can still be informative in equilibrium even when firms are asymmetric and offer asymmetric commissions, as long as the fraction of consumers that do not search more than once is large and the difference in the equilibrium commissions offered by firms is small.³¹ Given $\tau_1^* < \tau_2^*$ in equilibrium, steering by M causes (the less-efficient) firm 2 to capture a larger market share than it would have when M does not steer. This reinforces the fact that firm 1's market share is already too small in the absence of steering, due to the fact that cost differences are not fully passed through into price differences. Therefore, steering by M has an additional negative effect on welfare, in addition to the price level effect uncovered in the baseline model.

6.2 Fee-setting intermediary

We utilize our framework to explore the implications of steering in an alternative setup where rather than the firms setting commissions, M does. To fix ideas, it is useful to think of this setup as a stylized representation of an online platform where consumers can browse for products and perform transactions. Facing ex-ante identical firms, we focus on the case M sets a common commission (transaction fee) τ before firms' compete in prices. We continue to assume that consumers do not observe τ , but also note that the equilibrium we characterize would not change if they did. The relevant examples include online travel agents (OTAs) such as Expedia and Booking.com, and online marketplaces like the one run by Amazon. Similar to the baseline model, we focus on the symmetric informative equilibrium in which (i) all firms adopt the same strategy; (ii) M ranks all products in order of expected commission; and (iii) consumers inspect the highest ranked product without searching further given they believe that the highest-ranked product gives them the highest surplus.

In this setting, the only decision firms have to make is what price to charge after observing τ . It is then obvious that firms have no instrument to compete for M 's recommendation other than their price. Given M collects the same commission from all firms, it simply recommends the most suitable product (net of price) to non-shoppers. Steering plays no role in determining the equilibrium, and the pricing stage among firms reduces to the Perloff-Salop model. In this case, M sets τ as if it is a monopolist that sells a product with valuation $v + \max_{i=1,\dots,n} \{\epsilon_i\}$ and faces marginal cost equal to the firms' equilibrium markup. Let τ^p be the profit-maximizing commission

³⁰Note this model still coincides with the baseline model in Section 2 when $c_1 = c_2$.

³¹Related to this, Chakraborty and Harbaugh (2007) provide a general result that credible communication can be achieved through ranking in a reduced-form multidimensional cheap talk environment, provided the asymmetries between products are not too large.

set by M in this environment. The following proposition compares τ^P set by M versus τ^* set by firms in equilibrium:

Proposition 8 *Suppose M sets a common commission.*

1. *If $\bar{\epsilon} < \infty$, in the limit as n becomes large, $\lim_{n \rightarrow \infty} \tau^P = \lim_{n \rightarrow \infty} \tau^* = \tau^m$.*
2. *If F and G are linear, then $\tau^P > \tau^*$ for all finite $n \geq 2$.*

The first part of Proposition 8 states that M 's optimal commission when $n \rightarrow \infty$ is τ^m , which is the commission level that would be set by a continuum of firms when they can compete in commissions (Proposition 4). This result suggests that with a continuum of firms, the market outcome is the same regardless of the party that sets the commission. Even when firms set commissions, the intense market competition drives them to set a commission level that M would optimally set by itself. The second part of Proposition 8 suggests that for a finite number of firms, M will set a higher τ than the commission level set by the firms in equilibrium. From the perspective of firm i , additional recommendations from an increase in commission are beneficial only for instances where both $v + \epsilon_i - p_i$ and $v + \max_{j \neq i} \{\epsilon_j\} - p^*$ are greater than zero, as otherwise recommendations do not affect consumer purchase decisions. In contrast, from the perspective of M , an increase in the commission τ results in additional revenue from every inframarginal consumer, so that the benefit from increasing τ is realized as long as $\max_{i=1, \dots, n} \{v + \epsilon_i - p_i\} \geq 0$. Thus, the marginal benefit from an increase in τ is higher for M compared to individual firms. While the pass-through rate from τ to prices differs depending on whether M or firms set commissions, thus complicating the analysis, under linear F and G the result that M prefers a higher commission always holds.

6.3 Multiple intermediaries

Returning to the case in which firms compete in commissions and prices (e.g. the case of insurance brokers and financial advisors), we discuss how our benchmark analysis carries over when there are multiple information intermediaries. Given consumers are assumed to be uninformed about commissions and other variables that they could otherwise use to choose between intermediaries, their choice of a particular intermediary will be based on exogenous factors such as the location of the advisor. The remaining question is whether it would ever be rational for consumers to consider going to a rival intermediary to get a second set of rankings. If in equilibrium, firms affiliate with all intermediaries and all intermediaries recommend the best matched firm to consumers, there would be no reason to do so. This line of reasoning suggests the equilibrium outcome in our framework would remain an equilibrium in the presence of multiple intermediaries.

To illustrate our reasoning, consider m differentiated intermediaries $\{M_1, M_2, \dots, M_m\}$. Firms are able to costlessly affiliate with every intermediary (i.e., make their product available at the intermediary), and they can set different prices and commissions across intermediaries. For consumers, the idiosyncratic transportation (or shopping) cost associated with visiting an intermediary $k \in \{1, \dots, m\}$ is t_k , which is i.i.d drawn from a common distribution. Consumers visit intermediaries to obtain recommendations and to inspect and purchase products, and they can choose to incur the transportation cost multiple times to obtain recommendations from multiple intermediaries. A consumer knows nothing except her location before visiting any intermediary, and all

purchases are required to go through an intermediary. For simplicity, we assume that the market for intermediaries is fully covered so every consumer visits at least one intermediary.

In this environment, it is easy to verify that the informative equilibrium with steering in Proposition 1 remains a valid equilibrium where each firm sets price p^* and commission τ^* at each intermediary. Given that consumers expect each intermediary’s recommendation to be unbiased in equilibrium and that the price of each product is expected to be the same across all intermediaries, each consumer simply visits the intermediary that has the lowest realization of t_k (i.e., is nearest) without visiting any other intermediary. Once a consumer visits an intermediary, the recommendation and consumer search behaviour unfold as in the baseline model. Finally, given that consumers do not observe firms’ affiliation or pricing decisions before searching, and that consumers do not visit more than one intermediary in equilibrium, firms will want to affiliate with all intermediaries and set the same prices and commissions across intermediaries.

The reasoning above utilizes the fact that the only role of intermediaries from consumers’ perspective is to provide information and that consumers observe nothing before visiting intermediaries. Nonetheless, the same reasoning flows through even if we allow intermediaries to compete over some non-price dimensions to attract consumers, e.g., by advertising. Such non-price competition between intermediaries only affects which intermediary consumers go to without affecting how firms set prices and commissions.

7 Conclusion

By incorporating that both intermediaries and consumers have private information, this paper provides a new framework to study steering by an information intermediary, one in which firms compete in both prices and commissions to attract uninformed consumers.

In our framework, it is off-the-equilibrium path steering that drives the interesting market outcomes that we find. Because firms compete in commissions to be recommended, prices end up higher, and indeed firms, consumers and overall welfare end up being harmed by the possibility of steering. Surprisingly, we find greater firm competition results in higher commissions and higher prices. However, despite this, consumer surplus and welfare will be lower without the information intermediary, provided consumers face sufficiently high costs of search (or inspecting) products. We show the policies of absolute commission caps, commission disclosure, consumer education, and penalties for inappropriate advice can lower prices, and increase consumer surplus and welfare.

Our framework can usefully be extended in several major directions. In our current setup, consumers will rationally only inspect the top ranked product. While oftentimes consumers will indeed only evaluate the product recommended by their advisor, or the first ranked product on an online intermediary³², sometimes it seems reasonable that they may want to search beyond the recommended firm. We allowed for this possibility in Section 5 by either allowing some probability that consumers have zero search costs (i.e., are informed consumers) or some probability that consumers observe the commissions (so that they may search more than once if they observe commissions that are higher than expected). We also allowed for this possibility in Section 6, in

³²A case in point is Amazon’s buybox. Studies have found more than 80% of the time consumers do not search beyond the first ranked seller on Amazon, i.e., they only use the Buy Box (Chen et al., 2016).

the case there are two firms that have asymmetric costs, so that consumers will search beyond the high cost firm if it is recommended when their search cost is low enough.

Alternatively, we could have instead allowed for the possibility that consumers know something about their match value which cannot be communicated to the intermediary, or that the intermediary only imperfectly observes the consumer-product match values, thereby creating an incentive for consumers to sometimes want to continue searching even in the symmetric equilibrium. We believe the central intuition of our equilibrium construction, that the intermediary has incentives to provide surplus to consumers through informative recommendations given it wants to increase the chances that consumers buy a recommended product, should continue to hold in this environment. Provided the intermediary's information is not too noisy, it should still be optimal for consumers to search according to the ranking provided. However, pinning down the exact equilibrium in this environment is challenging as the search problem faced by consumers is generally non-stationary due to the interaction with the equilibrium recommendation. Such a possibility remains an interesting challenge for future research.

Finally, the provision of information about qualities (i.e., a vertical dimension) is another important role information intermediaries can provide that we, like most of the literature, abstracted from. Future research should explore the implications of steering in such a setting.

8 Appendix

8.1 A prior communication stage

In this appendix, we formally analyze a prior communication stage between M and consumers that occurs after firms set their prices and commissions but before M provides any recommendation. Specifically, suppose that for each $i = 1, \dots, n$, the consumer-product (random) match component is $\epsilon_i = y_i + z_i$, where the realization of y_i is privately observed by M while the realization of z_i is privately observed by consumers (before they inspect any product). We can allow for arbitrary, possibly interdependent, distributions for y_i and z_i as long as the resulting distribution of $y_i + z_i$ follows the common distribution F across all products. Consumers can costlessly send a non-verifiable message to M . The type of message is not restricted, but as will be clear below it suffices to focus on messages that are elements of the space of n -tuple $[0, \bar{\epsilon}]^n$, i.e., reports over the realized vector (z_1, \dots, z_n) . All other stages of the model unfold as described in Section 2.

In what follows, we show that the informative equilibrium with steering described in Proposition 1 can be sustained, with the following additional specifications on equilibrium strategies of consumers and M : (i) Each consumer truthfully reveals the realized value of (z_1, \dots, z_n) to M ; (ii) For each received message (z_1, \dots, z_n) , M ranks all products in order of $\tau_i(1 - G(p_i - y_i - z_i))$, expecting that consumers make truthful reports on (z_1, \dots, z_n) . Note that consumers do not observe the consumer-specific match component v_i prior to search, hence they cannot communicate that to M .

To verify the equilibrium, we first note that because equilibrium prices and commissions are the same across firms, M ranks products in the order of $y_i + z_i - p^*$ on the equilibrium path. This means that the equilibrium ex-ante surplus of a consumer from truthfully revealing (z_1, \dots, z_n) is

$$E \left[\max_{i=1, \dots, n} \{v + y_i + z_i - p^*, 0\} \right],$$

where the expectation is taken with respect to (y_1, \dots, y_n) . Regardless of how the consumer deviates in its message and what M does off-equilibrium (after receiving any non-equilibrium message from the consumer), the ex-ante surplus for the consumer must be bounded above by the surplus she gets in the ideal case where

M truthfully reveals all information it has so that the consumer becomes fully informed. The ex-ante surplus in this ideal case is $E[\max_{i=1,\dots,n}\{v + y_i + z_i - p^*, 0\}]$, which is exactly the equilibrium surplus for the consumer. Therefore, there is no strictly profitable deviation for the consumer from the proposed equilibrium. Finally, the recommendation strategy of M is clearly optimal given its belief over consumers' messages, and this belief is indeed consistent with the equilibrium strategy of consumers.

8.2 Proof of Proposition 1

The intermediary's and consumers' strategies in stages (ii) and (iii) are determined in Section 3.1. The relevant first-order conditions for inter-firm competition in stage (i) are

$$\frac{\partial \Pi_i}{\partial p_i} = (p_i - \tau_i) \frac{\partial D_i}{\partial p_i} + D_i = 0 \quad \text{and} \quad \frac{\partial \Pi_i}{\partial \tau_i} = (p_i - \tau_i) \frac{\partial D_i}{\partial \tau_i} - D_i = 0,$$

where

$$\begin{aligned} \frac{\partial D_i}{\partial p_i} &= - \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} f(\max\{\bar{x}_i(\epsilon), -v\} + p_i) dF(\epsilon)^{n-1} dG(v) < 0, \\ \frac{\partial D_i}{\partial \tau_i} &= - \int_{\underline{v}}^{\bar{v}} \int_{p^* - G^{-1}(1 - \frac{\tau_i}{\tau^*}(1 - G(v)))}^{\bar{\epsilon}} \left[\frac{d\bar{x}_i}{d\tau_i} f(\bar{x}_i(\epsilon) + p_i) \right] dF(\epsilon)^{n-1} dG(v) > 0. \end{aligned}$$

Note that the lower limit of the inner integration comes from the fact that changes in τ_i only matter when $-v < \bar{x}_i(\epsilon)$, or equivalently, $p^* - G^{-1}(1 - \frac{\tau_i}{\tau^*}(1 - G(-v)))$ by the definition of $\bar{x}_i(\epsilon)$. To obtain $\frac{d\bar{x}_i}{d\tau_i}$, we totally differentiate \bar{x}_i which gives $\frac{d\bar{x}_i}{d\tau_i} = -\frac{1}{\tau_i} \frac{1 - G(-\bar{x}_i(\epsilon))}{g(-\bar{x}_i(\epsilon))}$. Imposing symmetry, $p_i = p^*$ and $\tau_i = \tau^*$, we get

$$\begin{aligned} \frac{\partial D_i}{\partial p_i} &= - \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} f(\max\{\epsilon, p^* - v\}) dF(\epsilon)^{n-1} dG(v) < 0, \\ \frac{\partial D_i}{\partial \tau_i} &= \frac{1}{\tau^*} \int_{\underline{v}}^{\bar{v}} \int_{p^* - v}^{\bar{\epsilon}} \left[\frac{1 - G(p^* - \epsilon)}{g(p^* - \epsilon)} f(\epsilon) \right] dF(\epsilon)^{n-1} dG(v) > 0. \end{aligned}$$

Firm i 's demand in the symmetric equilibrium is

$$D_i = \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} [1 - F(\max\{\epsilon, p^* - v\})] dF(\epsilon)^{n-1} dG(v).$$

Rearranging the first-order conditions yields simultaneous equations (5) and (7). We relegate the verification of second-order conditions, which is rather lengthy, to Section A.6 of the Online Appendix.

We next establish existence and uniqueness of the equilibrium (p^*, τ^*) . Rewrite (5) and (7) as $p^* = \tau^* + \phi_1(p^*)$ and $\tau^* = \phi_2(p^*)$, where

$$\phi_1(p) \equiv \frac{\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} [1 - F(\max\{\epsilon, p - v\})] dF(\epsilon)^{n-1} dG(v)}{\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} f(\max\{\epsilon, p - v\}) dF(\epsilon)^{n-1} dG(v)} > 0 \quad (10)$$

$$\phi_2(p) \equiv \frac{\int_{\underline{v}}^{\bar{v}} \int_{p-v}^{\bar{\epsilon}} \left[\frac{1 - G(p - \epsilon)}{g(p - \epsilon)} f(\epsilon) \right] dF(\epsilon)^{n-1} dG(v)}{\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} f(\max\{\epsilon, p - v\}) dF(\epsilon)^{n-1} dG(v)} > 0. \quad (11)$$

Then, we simply need to show the existence of a unique solution to $p^* - \phi_1(p^*) - \phi_2(p^*) = 0$. In the subsequent formal claims below, we will prove ϕ_1 and ϕ_2 are both decreasing function. Using these two properties, by the intermediate value theorem, there exists a unique finite solution p^* in the compact interval $[0, \phi_1(0) + \phi_2(0)]$ as required.

To prove the claimed properties, let $\epsilon_{(n)}$ be the highest order statistic (out of n draws of ϵ).

Claim 1 $\phi_1(p)$ is decreasing in p .

Proof. Step 1.1: Denote a new random variable $\tilde{v} \equiv v - p$. Then the conditional random variable $\tilde{v}|_{\epsilon_{(n)} > -\tilde{v}}$, as pinned down by

$$\tilde{G}(x; p) \equiv \Pr(\tilde{v} < x | \epsilon_{(n)} > -\tilde{v}) = \frac{\int_{\underline{v}-p}^x (1 - F(-\tilde{v})^n) g(\tilde{v} + p) d\tilde{v}}{\int_{\underline{v}-p}^{\bar{v}-p} (1 - F(-\tilde{v})^n) g(\tilde{v} + p) d\tilde{v}} \quad \text{for } x \in [\underline{v} - p, \bar{v} - p], \quad (12)$$

is FOSD (first-order stochastic dominance) decreasing in p . To see this, by definition we need to show $\tilde{G}(x; p)$ is increasing in p at each given x . Taking the derivative of the CDF function, we have $\frac{\partial \tilde{G}(x; p)}{\partial p} \geq 0$ if

$$\frac{\int_{\underline{v}-p}^x [1 - F(-\tilde{v})^n] g'(\tilde{v} + p) d\tilde{v}}{\int_{\underline{v}-p}^x [1 - F(-\tilde{v})^n] g(\tilde{v} + p) d\tilde{v}} \geq \frac{\int_{\underline{v}-p}^{\bar{v}-p} [1 - F(-\tilde{v})^n] g'(\tilde{v} + p) d\tilde{v}}{\int_{\underline{v}-p}^{\bar{v}-p} [1 - F(-\tilde{v})^n] g(\tilde{v} + p) d\tilde{v}}. \quad (13)$$

Given $x \leq \bar{v} - p$, establishing (13) is equivalent to showing that the left-hand side of (13) is decreasing in x . If we define the distribution function

$$\begin{aligned} H_0(y; x) &= \Pr(\tilde{v} < y | -\epsilon_{(n)} < \tilde{v} < x) \\ &= \frac{\int_{\underline{v}-p}^y [1 - F(-\tilde{v})^n] g(\tilde{v} + p) d\tilde{v}}{\int_{\underline{v}-p}^x [1 - F(-\tilde{v})^n] g(\tilde{v} + p) d\tilde{v}} \quad \text{for } y \in [\underline{v} - p, x], \end{aligned}$$

then we can rewrite the left-hand side of (13) as $\int_{\underline{v}-p}^x \left[\frac{g'(y+p)}{g(y+p)} \right] dH_0(y; x)$. Log-concavity of g implies that $\frac{g'(y+p)}{g(y+p)}$ is decreasing in y . Meanwhile it is obvious from the definition that $H_0(y; x)$ is FOSD increasing in x . Therefore, we conclude that the left-hand side of (13) is decreasing in x so that inequality (13) indeed holds, implying that $\tilde{v}|_{\epsilon_{(n)} > -\tilde{v}}$ is indeed FOSD decreasing in p .

Step 1.2: Denote

$$\bar{\phi}_1(p - v) \equiv \frac{\frac{1}{n} (1 - F(p - v)^n)}{\int_{p-v}^{\bar{\epsilon}} [f(\epsilon)] dF(\epsilon)^{n-1} + f(p - v) F(p - v)^{n-1}}. \quad (14)$$

Lemma 4 of Zhou (2017) shows $\bar{\phi}_1(\cdot)$ is a decreasing function. Utilizing $\tilde{v} \equiv v - p$ and the fact that the (symmetric) equilibrium demand of each firm can be stated as

$$\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} [1 - F(\max\{\epsilon, p - v\})] dF(\epsilon)^{n-1} dG(v) = \frac{1}{n} \int_{\underline{v}}^{\bar{v}} [1 - F(p - v)^n] dG(v),$$

we can rewrite ϕ_1 as

$$\begin{aligned} \frac{1}{\phi_1(p)} &= \int_{\underline{v}}^{\bar{v}} \left[\frac{1}{\bar{\phi}_1(p - v)} \right] \left[\frac{(1 - F(p - v)^n)}{\int_{\underline{v}}^{\bar{v}} [1 - F(p - v)^n] dG(v)} \right] dG(v) \\ &= \int_{\underline{v}-p}^{\bar{v}-p} \left[\frac{1}{\bar{\phi}_1(-\tilde{v})} \right] d\tilde{G}(\tilde{v} | \epsilon_{(n)} > -\tilde{v}). \end{aligned}$$

Given $\frac{1}{\bar{\phi}_1(-\tilde{v})}$ is decreasing in \tilde{v} and \tilde{v} is FOSD decreasing in p (Step 1.1), we conclude $\frac{1}{\phi_1(p)}$ is increasing in p so that $\phi_1(p)$ is decreasing in p as required. ■

Claim 2 $\phi_2(p)$ is decreasing in p .

Proof. Step 2.1: Conditional random variable $\epsilon_{(n)}|_{\epsilon_{(n)} > p-v}$, as pinned down by

$$F_{(n)}(x|\epsilon_{(n)} > p-v) \equiv \Pr(\epsilon_{(n)} < x|\epsilon_{(n)} > p-v) = \frac{\int_{\underline{\epsilon}}^x [1-G(p-\epsilon)] dF(\epsilon)^n}{\int_{\underline{\epsilon}}^{\bar{\epsilon}} [1-G(p-\epsilon)] dF(\epsilon)^n} \quad \text{for } x \in [\underline{\epsilon}, \bar{\epsilon}],$$

is FOSD increasing in p . This can be proven with the same argument as in Step 1.1 by log-concavity of g .

Step 2.2: Denote a new random variable $\tilde{\epsilon} \equiv \epsilon_{(n)} - p$. Then the conditional random variable $\tilde{\epsilon}|_{\tilde{\epsilon} > -v}$, as pinned down by

$$\tilde{F}_{(n,p)}(x|\tilde{\epsilon} > -v) \equiv \Pr(\tilde{\epsilon} < x|\tilde{\epsilon} > -v) = \frac{\int_{\underline{\tilde{\epsilon}}}^x [1-G(-\tilde{\epsilon})] dF(\tilde{\epsilon}+p)^n}{\int_{\underline{\tilde{\epsilon}}}^{\bar{\tilde{\epsilon}}} [1-G(-\tilde{\epsilon})] dF(\tilde{\epsilon}+p)^n} \quad \text{for } x \in [\underline{\tilde{\epsilon}}-p, \bar{\tilde{\epsilon}}-p],$$

is FOSD decreasing in p . This can be proven with the same argument as in Step 1.1 by log-concavity of f .

Step 2.3: We first write

$$\begin{aligned} \frac{\phi_2(p)}{\phi_1(p)} &= \frac{\int_{\underline{v}}^{\bar{v}} \int_{p-v}^{\bar{\epsilon}} \left[\frac{1-G(p-\epsilon)}{g(p-\epsilon)} f(\epsilon) \right] dF(\epsilon)^{n-1} dG(v)}{\frac{1}{n} \int_{\underline{v}}^{\bar{v}} [1-F(p-v)]^n dG(v)} \\ &= \frac{\int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\frac{1-G(p-\epsilon)}{g(p-\epsilon)} f(\epsilon) (1-G(p-\epsilon)) \right] dF(\epsilon)^{n-1}}{\frac{1}{n} \int_{\underline{\epsilon}}^{\bar{\epsilon}} [1-G(p-\epsilon)] dF(\epsilon)^n} \\ &= (n-1) \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\frac{1-G(p-\epsilon)}{g(p-\epsilon)} \frac{f(\epsilon)}{F(\epsilon)} \right] dF_{(n)}(\epsilon|\epsilon_{(n)} > p-v) d\epsilon, \end{aligned} \quad (15)$$

where the second equality is obtained by changing the order of integration, while the third equality is due to $dF(\epsilon)^{n-1} = \frac{n-1}{n} \frac{1}{F(\epsilon)} dF(\epsilon)^n$ and the CDF definition of $\epsilon_{(n)}|_{\epsilon_{(n)} > p-v}$. Using integration by parts and variable substitution $\tilde{\epsilon}_{(n)} = \epsilon_{(n)} - p$, we get

$$\begin{aligned} \frac{1}{n-1} \left(\frac{\phi_2(p)}{\phi_1(p)} \right) &= \frac{1-G(p-\epsilon)}{g(p-\epsilon)} \frac{f(\bar{\epsilon})}{F(\bar{\epsilon})} \\ &\quad - \int_p^{\bar{\epsilon}} \left[\frac{1-G(p-\epsilon)}{g(p-\epsilon)} \left(\frac{f(\epsilon)}{F(\epsilon)} \right)' F_{(n)}(\epsilon|\epsilon_{(n)} > p-v) \right] d\epsilon \\ &\quad - \int_{\underline{\epsilon}}^{\bar{\epsilon}-p} \left[\left(\frac{1-G(-\tilde{\epsilon})}{g(-\tilde{\epsilon})} \right)' \frac{f(\tilde{\epsilon}+p)}{F(\tilde{\epsilon}+p)} \tilde{F}_{(n,p)}(\tilde{\epsilon}|\tilde{\epsilon}_{(n)} > -v) \right] d\tilde{\epsilon}, \end{aligned}$$

where $\left(\frac{f(\epsilon)}{F(\epsilon)} \right)' \leq 0$ and $\left(\frac{1-G(-\tilde{\epsilon})}{g(-\tilde{\epsilon})} \right)' \geq 0$ denote the derivative with respect to the function arguments. Taking derivatives and cancelling out common terms, we arrive at

$$\begin{aligned} \frac{1}{n-1} \frac{d}{dp} \left(\frac{\phi_2(p)}{\phi_1(p)} \right) &= - \int_p^{\bar{\epsilon}} \left[\frac{1-G(p-\epsilon)}{g(p-\epsilon)} \left(\frac{f(\epsilon)}{F(\epsilon)} \right)' \right] \underbrace{\frac{dF_{(n)}(\epsilon|\epsilon_{(n)} > p-v)}{dp}}_{\leq 0 \text{ (Step 2.1)}} d\epsilon \\ &\quad - \int_{\underline{\epsilon}}^{\bar{\epsilon}-p} \left[\left(\frac{1-G(-\tilde{\epsilon})}{g(-\tilde{\epsilon})} \right)' \frac{f(\tilde{\epsilon}+p)}{F(\tilde{\epsilon}+p)} \right] \underbrace{\frac{d\tilde{F}_{(n,p)}(\tilde{\epsilon}|\tilde{\epsilon}_{(n)} > -v)}{dp}}_{\geq 0 \text{ (Step 2.2)}} d\tilde{\epsilon} \\ &\leq 0, \end{aligned}$$

so that $\frac{\phi_2(p)}{\phi_1(p)}$ is decreasing in p . Together with Claim 1, we conclude $\phi_2(p) = \frac{\phi_2(p)}{\phi_1(p)} \phi_1(p)$ is decreasing in p . ■

8.3 Proof of Proposition 2 and 3

When there is no steering, each firm's demand is $\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} [1 - F(\max\{\epsilon_i, -v\} + p_i)] dF(\epsilon)^{n-1} dG(v)$, and the equilibrium price is given by $p' = \phi_1(p')$, where ϕ_1 as defined by (10). Given the equilibrium price with steering is $p^* = \tau^* + \phi_1(p^*)$, where $\tau^* > 0$, proving Proposition 2 amounts to showing $\frac{\partial p^*}{\partial \tau^*} > 0$. By the implicit function theorem and Claim 1, we know $\frac{\partial p^*}{\partial \tau^*} = \frac{1}{1 - d\phi_1/dp} \in (0, 1)$.

To prove Proposition 3, CS and W are clearly decreasing in p^* , and p^* is higher when M steers (from Proposition 2). For the result on $\sum \pi_i$, we already know $\frac{\partial p^*}{\partial \tau^*} \in (0, 1)$, so that the firms' equilibrium margin must be decreasing in τ^* , while the equilibrium level of sales for each firm is decreasing in τ^* as well. Since τ^* is higher when M steers, $\sum \pi_i$ must be lower when there is steering. Finally, since the commission is zero when there is no steering, an unbiased intermediary earns zero profit while a steering intermediary earns strictly positive profit.

8.4 Proof of Proposition 4

Recall from (10) and (11) that (p^*, τ^*) are pinned down by $p^* - \phi_1(p^*) - \phi_2(p^*) = 0$ and $\tau^* = \phi_2(p^*)$. We first prove the following two technical claims on the properties of functions $\phi_1(p; n)$ and $\phi_2(p; n)$.

Claim 3 (i) $\lim_{n \rightarrow \infty} \phi_1(p; n) = 0$ if $\bar{\epsilon} < \infty$; (ii) $\phi_1(p; n)$ is decreasing in n .

Proof. To prove the first part, write ϕ_1 in (10) as

$$\phi_1 = \frac{\frac{1}{n} \int_{\underline{v}}^{\bar{v}} [1 - F^n(p - v)] dG(v)}{\int_{\underline{v}}^{\bar{v}} [F^{n-1}(p - v) f(p - v)] dG(v) + \int_{\underline{v}}^{\bar{v}} \left[\int_{p-v}^{\bar{\epsilon}} f(\epsilon) dF(\epsilon)^{n-1} \right] dG(v)}. \quad (16)$$

The numerator clearly converges to zero. For the denominator, when $n \rightarrow \infty$ the distribution $F^{n-1}(\epsilon)$ converges to a distribution whose value is zero everywhere except at the distribution upperbound $\bar{\epsilon} < \infty$. Since G is also an atomless distribution, the first term in the denominator converges to zero. To find the limit value of the second term, let $x > 0$ be arbitrarily small. We have

$$\int_{p-v}^{\bar{\epsilon}} f(\epsilon) dF(\epsilon)^{n-1} = \int_{\bar{\epsilon}-x}^{\bar{\epsilon}} f(\epsilon) dF(\epsilon)^{n-1} + \int_{p-v}^{\bar{\epsilon}-x} f(\epsilon) dF(\epsilon)^{n-1}. \quad (17)$$

The second term of RHS in (17) converges to zero by the following comparison. Denote f_A^{\min} and f_A^{\max} as the lower and upper bounds of the density function within the closed interval A . Then

$$f_{[p-v, \bar{\epsilon}-x]}^{\min} \int_{v+p}^{\bar{\epsilon}-x} dF(\epsilon)^{n-1} < \int_{v+p}^{\bar{\epsilon}-x} f(\epsilon) dF(\epsilon)^{n-1} < f_{[p-v, \bar{\epsilon}-x]}^{\max} \int_{p-v}^{\bar{\epsilon}-x} dF(\epsilon)^{n-1}. \quad (18)$$

In (18), the first and the last term converges to zero because $\lim_{n \rightarrow \infty} F^{n-1}(\epsilon) = 0$ for all $\epsilon < \bar{\epsilon}$. By the squeeze theorem, the second term of (17) thus converges to zero. To find the limit value of the first term of RHS in (17), we apply the same technique:

$$f_{[\bar{\epsilon}-x, \bar{\epsilon}]}^{\min} \int_{\bar{\epsilon}-x}^{\bar{\epsilon}} dF(\epsilon)^{n-1} < \int_{\bar{\epsilon}-x}^{\bar{\epsilon}} f(\epsilon) dF(\epsilon)^{n-1} < f_{[\bar{\epsilon}-x, \bar{\epsilon}]}^{\max} \int_{\bar{\epsilon}-x}^{\bar{\epsilon}} dF(\epsilon)^{n-1}. \quad (19)$$

In (19), the first term converges to $f_{[\bar{\epsilon}-x, \bar{\epsilon}]}^{\min}$ while the last term converges to $f_{[\bar{\epsilon}-x, \bar{\epsilon}]}^{\max}$. Since x can be arbitrarily small and f is continuous, $f_{[\bar{\epsilon}-x, \bar{\epsilon}]}^{\min}$ and $f_{[\bar{\epsilon}-x, \bar{\epsilon}]}^{\max}$ converge to $f(\bar{\epsilon})$ for small enough x . Hence we can conclude that the term in the middle of (19) converges to $f(\bar{\epsilon})$. As a result, the second term in the denominator in equation (16) converges to $f(\bar{\epsilon}) > 0$, so that (16) indeed converges to zero.

To prove the second part, recall from the proof of Claim 1 that

$$\frac{1}{\phi_1} = \int_{\underline{v}-p}^{\bar{v}-p} \left[\frac{1}{\bar{\phi}_1(-\tilde{v}; n)} \right] d\tilde{G}(\tilde{v} | \epsilon_{(n)} > -\tilde{v}),$$

where $\bar{\phi}_1$ is defined in (14) and $\tilde{v} \equiv v - p$. Lemma 4 of Zhou (2017) shows $1/\bar{\phi}_1$ is increasing in n and decreasing in \tilde{v} . Hence, to conclude the claim, it remains to show that the random variable $\tilde{v}|_{\epsilon_{(n)} > -\tilde{v}}$ is FOSD decreasing in n . From CDF function (12), the relevant derivative $\frac{\partial \tilde{G}}{\partial n}$ can be shown to be positive if

$$\frac{\int_{\underline{v}-p}^x [-\ln F(-\tilde{v}) F(-\tilde{v})^n] g(\tilde{v} + p) d\tilde{v}}{\int_{\underline{v}-p}^x [1 - F(-\tilde{v})^n] g(\tilde{v} + p) d\tilde{v}} \leq \frac{\int_{\underline{v}-p}^{\bar{v}-p} [-\ln F(-\tilde{v}) F(-\tilde{v})^n] g(\tilde{v} + p) d\tilde{v}}{\int_{\underline{v}-p}^{\bar{v}-p} [1 - F(-\tilde{v})^n] g(\tilde{v} + p) d\tilde{v}}, \quad (20)$$

so that $\frac{\partial \tilde{G}}{\partial n} \geq 0$ if the left-hand side of (20) is increasing in x . Applying the same technique used in step 1.1 of the proof of Claim 1, we can write the left-hand side of (20) as $\int_{\underline{v}-p}^x \left[\frac{-\ln F(y) F(y)^n}{1 - F(y)^n} \right] dH_0(y; x)$. Since $-\ln F(y) \geq 0$, we know that $\frac{-\ln F(y) F(y)^n}{1 - F(y)^n}$ is increasing in y . This fact, together with the fact that H_0 is FOSD increasing in x , implies that left-hand side of (20) is increasing in x , so that inequality (20) indeed holds. Therefore, we conclude $\tilde{v}|_{\epsilon_{(n)} > -\tilde{v}}$ is FOSD decreasing in n , which proves the second part of the claim. \blacksquare

Claim 4 (i) $\lim_{n \rightarrow \infty} \phi_2(p; n) = \frac{1-G(p-\bar{\epsilon})}{g(p-\bar{\epsilon})}$ if $\bar{\epsilon} < \infty$; (ii) $\phi_2(p; n)$ and $\phi_1(p; n) + \phi_2(p; n)$ are increasing in n if condition (8) holds.

Proof. To prove the first part, note ϕ_2 has the same denominator as ϕ_1 , or (16). From Claim 3, the denominator converges to $f(\bar{\epsilon}) > 0$. To find the limit value of the numerator of ϕ_2 , note for an arbitrarily small $x > 0$, we have

$$\begin{aligned} & \int_{p-v}^{\bar{\epsilon}} \left[\frac{1-G(p-\epsilon)}{g(p-\epsilon)} f(\epsilon) \right] dF(\epsilon)^{n-1} \\ &= \int_{\bar{\epsilon}-x}^{\bar{\epsilon}} \left[\frac{1-G(p-\epsilon)}{g(p-\epsilon)} f(\epsilon) \right] dF(\epsilon)^{n-1} + \int_{p-v}^{\bar{\epsilon}-x} \left[\frac{1-G(p-\epsilon)}{g(p-\epsilon)} f(\epsilon) \right] dF(\epsilon)^{n-1}. \end{aligned} \quad (21)$$

Denote $\frac{(1-G)f}{g}|_A^{\min}$ and $\frac{(1-G)f}{g}|_A^{\max}$ as the lower and upper bound of the function $\frac{1-G(p-\epsilon)}{g(p-\epsilon)} f(\epsilon)$ for all ϵ within some closed interval A . Then the first term in (21) is bounded by

$$\frac{(1-G)f}{g}|_{[\bar{\epsilon}-x, \bar{\epsilon}]}^{\min} \int_{\bar{\epsilon}-x}^{\bar{\epsilon}} dF(\epsilon)^{n-1} \leq \int_{\bar{\epsilon}-x}^{\bar{\epsilon}} \left[\frac{1-G(p-\epsilon)}{g(p-\epsilon)} f(\epsilon) \right] dF(\epsilon)^{n-1} \leq \frac{(1-G)f}{g}|_{[\bar{\epsilon}-x, \bar{\epsilon}]}^{\max} \int_{\bar{\epsilon}-x}^{\bar{\epsilon}} dF(\epsilon)^{n-1}. \quad (22)$$

In (22), the first term converges to $\frac{(1-G)f}{g}|_{[\bar{\epsilon}-x, \bar{\epsilon}]}^{\min}$ while the last term converges to $\frac{(1-G)f}{g}|_{[\bar{\epsilon}-x, \bar{\epsilon}]}^{\max}$. Since x can be arbitrarily small and f is continuous, $\frac{Gf}{g}|_{[\bar{\epsilon}-x, \bar{\epsilon}]}^{\min}$ and $\frac{Gf}{g}|_{[\bar{\epsilon}-x, \bar{\epsilon}]}^{\max}$ converge to $\frac{G(\bar{\epsilon}-p)}{g(\bar{\epsilon}-p)} f(\bar{\epsilon})$ for small enough x . Hence we conclude that the middle term of (22) converges to $\frac{1-G(p-\bar{\epsilon})}{g(p-\bar{\epsilon})} f(\bar{\epsilon})$. Similarly, we can show that the second term in (21) converges to zero. Thus, the numerator of ϕ_2 converges to $\frac{1-G(p-\bar{\epsilon})}{g(p-\bar{\epsilon})} f(\bar{\epsilon})$, while the denominator converges to $f(\bar{\epsilon})$, which together yield the first part of the claim.

To prove the second part, denote denominator of ϕ_2 in (11) as $\chi \equiv \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} f(\max\{\epsilon, p-v\}) dF(\epsilon)^{n-1} dG(v)$. Since $f' \leq 0$ and the distribution of $F(\epsilon)^{n-1}$ is FOSD increasing in n , it follows that χ decreases with n . From the numerator of (11), changing the order of integration gives

$$\phi_2 = \frac{1}{\chi} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\frac{(1-G(p-\epsilon))^2}{g(p-\epsilon)} f(\epsilon) \right] (n-1) f(\epsilon) F(\epsilon)^{n-2}.$$

Using integration by parts,

$$\begin{aligned}\phi_2\chi &= \frac{(1-G(p-\bar{\epsilon}))^2}{g(p-\bar{\epsilon})}f(\bar{\epsilon}) \\ &\quad - \int_p^{\bar{\epsilon}} \left[\left(2 + \frac{(1-G(p-\epsilon))}{g(p-\epsilon)^2}g'(p^*-\epsilon) \right) + \frac{(1-G(p-\epsilon))}{g(p-\epsilon)}\frac{f'(\epsilon)}{f(\epsilon)} \right] (1-G(p-\epsilon))f(\epsilon)F^{n-1}(\epsilon)d\epsilon.\end{aligned}$$

The first term is constant in n , while the bracket term in the integral term can be written as

$$1 + \frac{\partial}{\partial v} \left(\frac{1-G(v)}{g(v)} \right) \Big|_{v=p-\epsilon} + \frac{(1-G(p-\epsilon))}{g(p-\epsilon)}\frac{f'(\epsilon)}{f(\epsilon)} \geq 1,$$

where the inequality is due to condition (8), given $p-\epsilon+\epsilon \geq 0$. Therefore, the integral term is decreasing in n because $F^{n-1}(\epsilon)$ is decreasing in n , and it follows that $\phi_2\chi$ is increasing in n . Then, given both ϕ_2 and χ are positive, we conclude ϕ_2 is increasing in n .

Next, the numerator of (10) can be rearranged and simplified as $\int_{\bar{\epsilon}}^{\bar{\epsilon}} [1-G(p-\epsilon)]f(\epsilon)F^{n-1}(\epsilon)d\epsilon$. Hence

$$\begin{aligned}(\phi_1 + \phi_2)\chi &= \frac{(1-G(p-\bar{\epsilon}))^2}{g(p-\bar{\epsilon})}f(\bar{\epsilon}) \\ &\quad - \int_p^{\bar{\epsilon}} \left[\left(1 + \frac{(1-G(p-\epsilon))}{g(p-\epsilon)^2}g'(p^*-\epsilon) \right) + \frac{(1-G(p-\epsilon))}{g(p-\epsilon)}\frac{f'(\epsilon)}{f(\epsilon)} \right] (1-G(p-\epsilon))f(\epsilon)F^{n-1}(\epsilon)d\epsilon.\end{aligned}$$

Again, condition (8) implies $(\phi_1 + \phi_2)\chi$ is increasing in n , and so does $\phi_1 + \phi_2$. ■

Completing the proof of Proposition 4.

From $p^* = \tau^* + \phi_1(p^*)$, Claims 3.i and 4.i imply

$$\lim_{n \rightarrow \infty} p^* = \tau^* = \lim_{n \rightarrow \infty} \phi_2(p^*) = \frac{1-G(p^*-\bar{\epsilon})}{g(p^*-\bar{\epsilon})},$$

so that the limit equilibrium price is exactly τ^m by definition, which proves Proposition 4.1. To prove Proposition 4.2, using the implicit function theorem we get

$$\frac{dp^*}{dn} = \frac{\partial\phi_1/\partial n + \partial\phi_2/\partial n}{1 - \partial\phi_1/\partial p^* - \partial\phi_2/\partial p^*} \geq 0,$$

where the numerator is positive by Claim 4.ii, while the denominator is positive by Claims 1-2. Likewise,

$$\begin{aligned}\frac{d\tau^*}{dn} &= \frac{\partial\phi_2}{\partial n} + \frac{\partial\phi_2}{\partial p^*}\frac{dp^*}{dn} \\ &= \frac{\partial\phi_2}{\partial n} + \frac{\partial\phi_2}{\partial p^*} \left(\frac{\partial\phi_1/\partial n + \partial\phi_2/\partial n}{1 - \partial\phi_1/\partial p^* - \partial\phi_2/\partial p^*} \right) \\ &= \frac{\partial\phi_2}{\partial n} \left(1 - \frac{-\partial\phi_2/\partial p^*}{1 - \partial\phi_1/\partial p^* - \partial\phi_2/\partial p^*} \right) + \frac{\partial\phi_2}{\partial p^*} \left(\frac{\partial\phi_1/\partial n}{1 - \partial\phi_1/\partial p^* - \partial\phi_2/\partial p^*} \right) \geq 0,\end{aligned}$$

where the last inequality utilizes $\frac{-\partial\phi_2/\partial p^*}{1 - \partial\phi_1/\partial p^* - \partial\phi_2/\partial p^*} \leq 1$, and Claims 3.ii - 4.ii.

8.5 Proof of Proposition 5

In Section A.9 of the online appendix, we derive the optimal search strategy of consumers and obtain the following demand function for each firm i

$$D_i(p_i) = \int_{\min\{\bar{p}-\bar{x}, \bar{v}\}}^{\bar{v}} \left[\frac{1-F(\bar{x}-\bar{p}+p_i)}{1-F(\bar{x})} \right] dG(v) + \int_{\bar{v}}^{\min\{\bar{p}-\bar{x}, \bar{v}\}} [1-F(p_i-v)] dG(v), \quad (23)$$

where \tilde{p} is the symmetric equilibrium price set by other firms, and \tilde{x} is the reservation value defined as the solution to $\int_{\tilde{x}}^{\bar{\epsilon}} [\epsilon - \tilde{x}] dF(\epsilon) = s$. Firm i has profit function $\Pi_i = p_i D_i(p_i)$. Assuming that the second-order condition holds, a sufficient condition of which is for both F and G to be linear, we can solve for the first-order condition and apply symmetry to obtain the equilibrium price

$$\tilde{p} = \frac{1 - G(\min\{\tilde{p} - \tilde{x}, \bar{v}\}) + \int_{\underline{v}}^{\min\{\tilde{p} - \tilde{x}, \bar{v}\}} [1 - F(\tilde{p} - v)] dG(v)}{\frac{f(\tilde{x})}{1 - F(\tilde{x})} (1 - G(\min\{\tilde{p} - \tilde{x}, \bar{v}\})) + \int_{\underline{v}}^{\min\{\tilde{p} - \tilde{x}, \bar{v}\}} f(\tilde{p} - v) dG(v)}. \quad (24)$$

Given the supposition that F and G are linear with distribution support $[-1, 1]$, the equilibrium price can be explicitly solved as

$$\tilde{p} = \begin{cases} \frac{1}{4} (5 - (2 - \tilde{x})\tilde{x} - \sqrt{\tilde{x}^4 + 18\tilde{x}^2 - 8\tilde{x} + 5}) & \text{if } \tilde{x} \geq -\frac{1}{3} \\ \frac{2}{3} & \text{if } \tilde{x} < -\frac{1}{3} \end{cases},$$

where \tilde{p} is continuous in \tilde{x} , and we have $\tilde{x} \geq -1/3$ if and only if $s \leq 4/9$.

Consumer surplus is the sum of surpluses of consumers who search exactly once (those with $v < \tilde{p} - \tilde{x}$) and consumers who search more than once (those with $v \geq \tilde{p} - \tilde{x}$). Therefore, if the search market is active, consumer surplus is

$$\widetilde{CS} = \int_{-1}^{\tilde{p} - \tilde{x}} \int_{\tilde{p} - v}^1 [v + \epsilon - \tilde{p}] d\epsilon dv + \int_{\tilde{p} - \tilde{x}}^1 \int_{-1}^1 [v + \max\{\tilde{x}, \epsilon\} - \tilde{p}] d\epsilon dv - s,$$

while the welfare is

$$\widetilde{W} = \int_{-1}^{\tilde{p} - \tilde{x}} \int_{\tilde{p} - v}^1 [v + \epsilon] d\epsilon dv + \int_{\tilde{p} - \tilde{x}}^1 \int_{-1}^1 [v + \max\{\tilde{x}, \epsilon\}] d\epsilon dv - s.$$

If the market is inactive and all consumers buy the outside option, we have $\widetilde{CS} = \widetilde{W} = 0$. We first note that the search market is inactive for all $s > 4/9$ (so that $\tilde{x} < -1/3$): if the market were active then $\tilde{p} - \tilde{x} > 1$ and

$$\begin{aligned} \widetilde{CS} &= \int_{-1}^{\tilde{p} - \tilde{x}} \int_{\tilde{p} - v}^1 [\epsilon + v - \tilde{p}] d\epsilon dv + \int_{\tilde{p} - \tilde{x}}^1 \int_{-1}^1 [\max\{\tilde{x}, \epsilon\} + v - \tilde{p}] d\epsilon dv - s \\ &= \int_{-1}^1 \int_{\tilde{p} - v}^1 [\epsilon + v - \tilde{p}] d\epsilon dv - s = \frac{5}{9} \left(\frac{4}{9} \right) - \frac{4}{9} < 0. \end{aligned}$$

So, it suffices to focus on the case $s \leq 4/9$. From direct calculations, one can verify that (i) $\widetilde{CS} < 0$ for all $s > 1/4$, and (ii) \tilde{p} is increasing in s for all $s \in [0, 1/4]$. Utilizing (ii), a total differentiation on \widetilde{CS} shows $\frac{d\widetilde{CS}}{ds} < -1$ for all $s \in [0, 1/4]$, so that intermediate value theorem and (i) implies the existence of a unique threshold $\bar{s}_0 \in [0, 1/4]$ above which the search market without intermediation is inactive. Moreover, (ii) implies \tilde{p} is maximized at $s = 1/4$, whereby $\tilde{p} = \frac{5 - \sqrt{5}}{4} < 1 = p^*$, i.e. the equilibrium price with intermediation and steering.

To establish threshold \bar{s}_1 , we first note that $\widetilde{CS} - CS = 1 - \frac{1}{4} > 0$ if $s \rightarrow 0$, and $\widetilde{CS} - CS = 0 - 0 = 0$ if $s = \frac{1}{4}$. In addition, given that the market outcome with intermediation is independent of s as long as $CS \geq 0$, we have $\frac{d}{ds} (\widetilde{CS} - CS) = \frac{d\widetilde{CS}}{ds} + 1 < 0$. Intermediate value theorem then implies the existence of the unique threshold $\bar{s}_1 \in [0, 1/4]$. As for welfare, we have $\widetilde{W} - W = 1 - (\frac{1}{4} + \frac{1}{2}) > 0$ if $c \rightarrow 0$, $\widetilde{W} - W = 0 - \frac{1}{2} < 0$ if $s = \frac{1}{4}$, and $\frac{d}{ds} (\widetilde{W} - W) = \frac{d\widetilde{W}}{ds} + 1 < 0$, so that Intermediate value theorem implies the existence of the unique threshold $\bar{s}_2 \in [0, 1/4]$. Finally, the fact that $p^* = 1$ (the $\bar{\epsilon}$ -monopoly price level) implies that the joint profit of firms and intermediary is maximized in the presence of intermediary, i.e., $\widetilde{W} - \widetilde{CS} \geq W - CS$. Rearranging the inequality shows $\widetilde{W} - W \geq \widetilde{CS} - CS$ for all s , which then implies $\bar{s}_2 \leq \bar{s}_1$.

References

- Anderson, S. P., A. de Palma, and J. F. Thisse (1992). *Discrete choice theory of product differentiation*. MIT press.
- Anderson, S. P. and R. Renault (1999). Pricing, product diversity, and search costs: A Bertrand-Chamberlin-Diamond model. *RAND Journal of Economics*, 719–735.
- Armstrong, M. and J. Zhou (2011). Paying for prominence. *The Economic Journal* 121(556).
- Armstrong, M. and J. Zhou (2019). Consumer information and the limits to competition. *Working Paper*.
- Bagnoli, M. and T. Bergstrom (2005). Log-concave probability and its applications. *Economic theory* 26(2), 445–469.
- Caplin, A. and B. Nalebuff (1991). Aggregation and imperfect competition: On the existence of equilibrium. *Econometrica*, 25–59.
- Chakraborty, A. and R. Harbaugh (2007). Comparative cheap talk. *Journal of Economic Theory* 132(1), 70–94.
- Chater, N., S. Huck, and R. Inderst (2010). Consumer decision-making in retail investment services: A behavioural economics perspective. *Report to the European Commission/SANCO*.
- Chen, L., A. Mislove, and C. Wilson (2016). An empirical analysis of algorithmic pricing on Amazon marketplace. In *Proceedings of the 25th International Conference on World Wide Web*, pp. 1339–1349. International World Wide Web Conferences Steering Committee.
- Chen, Y. and C. He (2011). Paid placement: Advertising and search on the internet. *The Economic Journal* 121(556), F309–F328.
- Crawford, V. P. and J. Sobel (1982). Strategic information transmission. *Econometrica*, 1431–1451.
- de Cornière, A. and G. Taylor (2014). Integration and search engine bias. *RAND Journal of Economics* 45(3), 576–597.
- de Cornière, A. and G. Taylor (2019). A model of biased intermediation. *RAND Journal of Economics forthcoming*.
- Edelman, B. and J. Wright (2015). Markets with price coherence. *HBS Working Paper*.
- Eliasz, K. and R. Spiegel (2011). A simple model of search engine pricing. *The Economic Journal* 121(556), F329–F339.
- Engelberg, J., C. Parsons, and N. Tefft (2014). Financial conflicts of interest in medicine. *Working Paper*.
- Hagiu, A. and B. Jullien (2011). Why do intermediaries divert search? *RAND Journal of Economics* 42(2), 337–362.
- Hockenberry, J. M., P. Weigel, A. Auerbach, and P. Cram (2011). Financial payments by orthopedic device makers to orthopedic surgeons. *Archives of internal medicine* 171(19), 1759–1765.
- Hunold, M. and J. Muthers (2017). Resale price maintenance and manufacturer competition for retail services. *RAND Journal of Economics* 48(1), 3–23.

- Hunter, R. J. (2006). Title insurance cost and competition. *Testimony Before the House Committee on Financial Services - Subcommittee on Housing and Community Opportunity*.
- In, Y. and J. Wright (2018). Signaling private choices. *Review of Economic Studies* 85(1), 558–580.
- Inderst, R. (2015). Regulating commissions in markets with advice. *International Journal of Industrial Organization* 43, 137–141.
- Inderst, R. and M. Ottaviani (2012a). Competition through commissions and kickbacks. *American Economic Review* 102(2), 780–809.
- Inderst, R. and M. Ottaviani (2012b). How (not) to pay for advice: A framework for consumer financial protection. *Journal of Financial Economics* 105(2), 393–411.
- Lacko, J. M. and J. K. Pappalardo (2004). The effect of mortgage broker compensation disclosures on consumers and competition: A controlled experiment. Technical report, Federal Trade Commission Bureau of Economics.
- McAfee, R. P. and M. Schwartz (1994). Opportunism in multilateral vertical contracting: Nondiscrimination, exclusivity, and uniformity. *American Economic Review*, 210–230.
- Oxera (2015). Regulating remuneration systems: effective distribution of financial products. Technical report, Oxera Consulting LLP.
- Perloff, J. M. and S. C. Salop (1985). Equilibrium with product differentiation. *Review of Economic Studies* 52(1), 107–120.
- Prékopa, A. (1971). Logarithmic concave measures with application to stochastic programming. *Acta Scientiarum Mathematicarum* 32, 301–316.
- Prékopa, A. (1973). On logarithmic concave measures and functions. *Acta Scientiarum Mathematicarum* 34, 335–343.
- Thiel, J. (2019). The remuneration of advisers and the advice gap. *Working Paper*.
- White, A. (2013). Search engines: Left side quality versus right side profits. *International Journal of Industrial Organization* 31(6), 690–701.
- Wolinsky, A. (1986). True monopolistic competition as a result of imperfect information. *The Quarterly Journal of Economics* 101(3), 493–511.
- Zhou, J. (2017). Competitive bundling. *Econometrica* 85(1), 145–172.

For Online Publication: Supplementary Appendix

Tat-How Teh* and Julian Wright†

In Section A of this online appendix, we provide proofs of omitted results and details from the baseline model of the main paper. In Section B we provide the formal analysis and more detailed results for Section 5 in the main paper (i.e., the policy implications), and in Section C we do likewise for Section 6 (i.e., the extensions).

A Further results of the baseline model

A.1 Sub-optimality of full disclosure by M

In the main text, we restricted M 's communication space to product rankings. A natural question is whether M has an incentive to reveal everything to consumers (full disclosure) if it is able to do so. Based on the same equilibrium described in Section 3, in what follows we argue that M cannot do better by deviating to full disclosure.

Obviously, full disclosure does not affect the on-equilibrium path outcome. In the equilibrium, all firms offer symmetric commissions and prices, so M ranks product j' first if and only if $j' = \arg \max_{j'=1,\dots,n} \{\epsilon_{j'} - p^*\}$, and consumers will buy the top-ranked product or the outside option. If M reveals all information, consumers would not change their decision.

Consider instead the off-equilibrium path in which some firms deviate by charging off-equilibrium prices and commissions. By ranking products according to expected commissions, M 's expected profit is

$$\Pi = \max_{j=1,\dots,n} \{\tau_j (1 - G(p_j - \epsilon_j))\}$$

given that consumers always buy the top-ranked product. By fully disclosing all information, consumers will buy the highest-surplus product instead, and the expected profit to M is

$$\Pi' = \tau_{j'} (1 - G(p_{j'} - \epsilon_{j'})) \text{ where } j' = \arg \max_{j'=1,\dots,n} \{\epsilon_{j'} - p_{j'}\}$$

Clearly, $\Pi \geq \Pi'$ because the definition of maximization implies

$$\max_{j=1,\dots,n} \{\tau_j (1 - G(p_j - \epsilon_j))\} \geq \tau_{j'} (1 - G(p_{j'} - \epsilon_{j'})).$$

A.2 General message space

The derived *informative equilibrium with steering* in the paper remains an equilibrium (PBE) in a game with a general message space. Let S be the general message space and \bar{S} be the set of all messages based on M providing a ranking. Given an equilibrium in which only \bar{S} is used, we can construct a new equilibrium in which all the messages in S are used and the outcome remains the same. Let N denote the number of messages in \bar{S} . Partition S/\bar{S} into N subsets, and let S_i denote the i -th subset in the partition of S/\bar{S} and \bar{s}_i denote the i -th message in \bar{S} .

Now, in the new equilibrium, whenever M would have sent $\bar{s}_i \in \bar{S}$ in the original equilibrium, it now sends any message $\{\bar{s}_i\} \cup S_i$ (or uses a mixed message with the subset as a support). As each consumer's inference after receiving a message will be the same as in the original equilibrium, M would use this strategy, and this is an equilibrium with a general message S .

*Department of Economics, National University of Singapore, E-mail: tehtathow@u.nus.edu

†Department of Economics, National University of Singapore, E-mail: jwright@nus.edu.sg

A.3 Wary beliefs

To characterize the informative equilibrium with steering in the main text, we have specified that consumers hold *passive beliefs* over unobserved commissions, as is commonly assumed in the literature on vertical contracting. An alternative approach also analyzed by that literature is called wary beliefs (McAfee and Schwartz, 1994), which has been generalized by In and Wright (2018). When observing a deviating contract offered from a common manufacturer, a retailer that holds wary beliefs will try to infer how the manufacturer should have optimally (and secretly) adjusted contracts offered to other, competing downstream retailers. In what follows, we show that our equilibrium characterization in Proposition 1 remains valid under wary beliefs, provided F and G are linear.

A consumer, after inspecting a product i and observing an off-equilibrium price $p_i \neq p^*$, tries to infer how firm i , in anticipation of this deviation, should have optimally adjusted its commission τ_i . When firms still expect the consumers to follow M 's recommendation, we will show that given F and G are linear, an individual firm's optimal τ_i is independent of p_i . In other words, consumers are unable to infer anything new about τ_i from the observed p_i . Consequently, under wary beliefs, consumers continue to infer that M , whose recommendation strategy remains described by (1), is recommending the highest-surplus product. Therefore, it is indeed optimal for consumers to only search the recommended product without searching further.

Recall that a deviant firm i 's first-order condition for optimal commission τ_i is

$$\frac{\partial D_i}{\partial \tau_i} - \frac{\partial D_i}{\partial p_i} = 0.$$

Suppose F and G are linear over $[\underline{\epsilon}, \bar{\epsilon}]$ and $[\underline{v}, \bar{v}]$, we have

$$\begin{aligned} \bar{x}_i(\epsilon) &= -G^{-1} \left(1 - \frac{\tau^*}{\tau_i} (1 - G(p^* - \epsilon)) \right) \\ &= - \left[(\bar{v} - \underline{v}) \left(1 - \frac{\tau^*}{\tau_i} \left(1 - \left(\frac{p^* - \epsilon - \underline{v}}{\bar{v} - \underline{v}} \right) \right) \right) + \underline{v} \right] \\ &= \frac{\tau^*}{\tau_i} (\bar{v} + \epsilon - p^*) - \bar{v}. \end{aligned}$$

The demand function and its derivatives are given by

$$\begin{aligned} D_i &= \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[1 - \frac{(\max\{\bar{x}_i(\epsilon), -v\}) + p_i - \epsilon}{\bar{\epsilon} - \underline{\epsilon}} \right] \left(\frac{n-1}{\bar{\epsilon} - \underline{\epsilon}} \right) \left(\frac{\epsilon - \underline{\epsilon}}{\bar{\epsilon} - \underline{\epsilon}} \right)^{n-2} \left(\frac{1}{\bar{v} - \underline{v}} \right) d\epsilon dv \\ \frac{\partial D_i}{\partial p_i} &= - \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\frac{1}{\bar{\epsilon} - \underline{\epsilon}} \right] \left(\frac{n-1}{\bar{\epsilon} - \underline{\epsilon}} \right) \left(\frac{\epsilon - \underline{\epsilon}}{\bar{\epsilon} - \underline{\epsilon}} \right)^{n-2} \left(\frac{1}{\bar{v} - \underline{v}} \right) d\epsilon dv < 0, \\ \frac{\partial D_i}{\partial \tau_i} &= \int_{\underline{v}}^{\bar{v}} \int_{\frac{\tau_i}{\tau^*}(\bar{v}-v)+p^*-\bar{v}}^{\bar{\epsilon}} \left[\frac{\tau_j}{\tau_i^2} (\bar{v} + \epsilon - p_j) \left(\frac{1}{\bar{\epsilon} - \underline{\epsilon}} \right) \right] \left(\frac{n-1}{\bar{\epsilon} - \underline{\epsilon}} \right) \left(\frac{\epsilon - \underline{\epsilon}}{\bar{\epsilon} - \underline{\epsilon}} \right)^{n-2} \left(\frac{1}{\bar{v} - \underline{v}} \right) d\epsilon dv > 0. \end{aligned}$$

Let $\hat{\tau}_i$ denote, for each p_i , the optimal commission defined by the first-order condition $\frac{\partial D_i}{\partial \tau_i} = -\frac{\partial D_i}{\partial p_i}$. As per the standard supermodularity argument, using the implicit function theorem we can pin down how $\hat{\tau}_i$ changes with p_i as follows:

$$\frac{\partial \hat{\tau}_i}{\partial p_i} = - \left(\frac{\frac{\partial^2 D_i}{\partial p_i \partial \tau_i} - \frac{\partial^2 D_i}{\partial p_i^2}}{\frac{\partial^2 D_i}{\partial \tau_i^2} - \frac{\partial^2 D_i}{\partial p_i \partial \tau_i}} \right).$$

Crucially, the linearity of demand implies $\frac{\partial^2 D_i}{\partial p_i \partial \tau_i} = 0$ and $\frac{\partial^2 D_i}{\partial p_i^2} = 0$. Meanwhile, it is easily verified that $\frac{\partial^2 D_i}{\partial \tau_i^2} < 0$. Therefore $\frac{\partial \hat{\tau}_i}{\partial p_i} = 0$, meaning that firm i 's optimal commission does not depend on the price it sets.

A.4 Steering with commitment

In the main text we assumed M cannot commit to its recommendation rule. While this non-commitment assumption fits our primary motivating examples of financial and insurance brokers, one may nonetheless be interested in what happens when M can announce and commit to specific recommendation rules before firms set prices and commissions. In particular, this means that M 's recommendation is no longer required to be sequentially rational. All other aspects of the model follow our baseline model in Section 2 of the main text.

We first note that among all possible recommendation rules, the upper bound to the profit achievable by M is the maximized joint-industry profit, that is

$$\widehat{\Pi} \equiv \max_p \left\{ p \int_{\underline{\epsilon}}^{\bar{\epsilon}} [(1 - G(p - \epsilon))] dF^n(\epsilon) \right\}. \quad (\text{A.1})$$

The possibility of price discrimination is ruled out because firms set their price before M observes consumer match values. With the constraint of uniform pricing by firms, the highest achievable profit is then exactly $\widehat{\Pi}$. Here, $\widehat{\Pi}$ is the same profit as obtained by a monopolist that sells a product with valuation $v + \max_{i=1, \dots, n} \{\epsilon_i\}$ to consumers directly assuming consumers are fully informed.

It turns out that M can exactly achieve profit $\widehat{\Pi}$ by committing to (i) recommend the highest commission product subject to the price cap \bar{p} ; and (ii) when there are multiple products with the highest commission, M breaks ties in favor of the product with the highest surplus. To see why the price cap is necessary, suppose first that M commits to recommending the highest commission product. Each firm always has an incentive to slightly increase its level of commission provided it earns a positive margin, so as to attract the entire market. The standard Bertrand logic implies that in the resulting equilibrium, a typical firm i will set its commission at $\tau_i = p_i$ such that it earns a zero margin. Crucially, however, p_i is a choice variable so firm i can always profitably simultaneously slightly increase p_i and increase τ_i by almost the same amount to beat its rival in the competition for recommendations, and earn a positive margin, as long as the chosen p_i still leads to a positive demand (i.e., that there are realizations such that $v + \epsilon_i - p_i \geq 0$). Given the distribution support of ϵ_i and v , firm i will want to keep raising its price (and commission) until $p_i = \bar{v} + \bar{\epsilon}$. Hence, the only possible equilibrium outcome is one where all firms set $\tau_i = p_i = \bar{v} + \bar{\epsilon}$. This is clearly an undesirable outcome for M because all consumers will prefer the outside option except when the match value realization is such that $\epsilon_i = \bar{\epsilon}$ and $v = \bar{v}$, which is a zero probability event.

In contrast, with the addition of a price cap, the outcome would be firms all set commissions and prices at the level of the imposed price cap. This means that M can use its price cap to implement any desired final product price, in particular, the price that is associated with (A.1).¹ Moreover, given that all firms are offering the same commission in equilibrium, M provides an unbiased recommendation to consumers so that consumers continue to believe that M 's recommendation is informative. To summarize:

Proposition 9 *Suppose M can credibly commit to always recommending the highest commission product subject to a price cap \bar{p} , and it breaks ties in favor of the product with the highest surplus. Then it can obtain profit $\widehat{\Pi}$, which is the highest possible profit that M can achieve among all possible recommendation rules.*

Under steering with commitment, the resulting final price is as if there is a single monopoly selling a product with valuation $v + \max_{i=1, \dots, n} \{\epsilon_i\}$, i.e. M 's optimal price cap solves (A.1). Comparing this price level to the resulting price without steering, the following proposition, which is analogous to Propositions 2 - 4, shows that all our insights on the implications of steering in Section 4 remain valid even when M has commitment power.

¹The outcome is similar to that arising when M sets a common per-transaction fee on all firms, as shown in Section 6.2. Here, however, because such a price-cap plus commitment eliminates the firms' margins in equilibrium, it delivers the highest possible profit to M .

Proposition 10 *When M can steer with commitment:*

1. \bar{p} is increasing in n . If $\bar{\epsilon} < \infty$ then $\lim_{n \rightarrow \infty} \bar{p} \rightarrow \tau^m$.
2. The level of prices and commissions as well as Π_M are higher than the equilibrium without steering.
3. CS , $\sum \pi_i$, and W are lower than the equilibrium without steering.
4. Proposition 5 still holds.

Proof. Rewrite and expand the definition of \bar{p} as

$$\bar{p} = \arg \max_p \left\{ p \int_{\underline{\epsilon}}^{\bar{\epsilon}} [(1 - G(p - \epsilon)) n f(\epsilon) F^{n-1}(\epsilon)] d\epsilon \right\}.$$

Given that log-concavity is preserved by multiplication, the log-concavity assumption on density functions ensures that the integrand is log-concave. Therefore, the demand function is log-concave because log-concavity is preserved by integration. Consequently, \bar{p} can be pinned down by first-order condition:

$$\frac{1}{\bar{p}} = \frac{\int_{\underline{\epsilon}}^{\bar{\epsilon}} [g(\bar{p} - \epsilon)] dF^n(\epsilon)}{\int_{\underline{\epsilon}}^{\bar{\epsilon}} [1 - G(\bar{p} - \epsilon)] dF^n(\epsilon)}. \quad (\text{A.2})$$

Log-concavity of demand ensures that the right-hand side of (A.2) is decreasing in \bar{p} . To establish $\frac{d\bar{p}}{dn} \geq 0$ it remains to show the right-hand side of (A.2) is decreasing in n . We have

$$\begin{aligned} \frac{\int_{\underline{\epsilon}}^{\bar{\epsilon}} [g(\bar{p} - \epsilon)] dF^n(\epsilon)}{\int_{\underline{\epsilon}}^{\bar{\epsilon}} [1 - G(\bar{p} - \epsilon)] dF^n(\epsilon)} &= \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\frac{g(\bar{p} - \epsilon)}{1 - G(\bar{p} - \epsilon)} \right] \frac{1 - G(\bar{p} - \epsilon)}{\int_{\underline{\epsilon}}^{\bar{\epsilon}} [1 - G(\bar{p} - \epsilon)] dF^n(\epsilon)} dF^n(\epsilon) \\ &= \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\frac{g(\bar{p} - x)}{1 - G(\bar{p} - x)} \right] dF_{(n)}(x | \epsilon_{(n)} > \bar{p} - v), \end{aligned} \quad (\text{A.3})$$

where $F_{(n)}(x | \epsilon_{(n)} > \bar{p} - v)$ is the CDF of the highest-order statistic $\epsilon_{(n)}$ (out of n i.i.d draws on ϵ), conditioned on the highest-order statistic being greater than $\bar{p} - v$:

$$\begin{aligned} F_{(n)}(x | \epsilon_{(n)} > \bar{p} - v) &= \Pr(\epsilon_{(n)} < x | \epsilon_{(n)} > \bar{p} - v) \\ &= \frac{\int_{\underline{\epsilon}}^x [1 - G(\bar{p} - \epsilon)] dF^n(\epsilon)}{\int_{\underline{\epsilon}}^{\bar{\epsilon}} [1 - G(\bar{p} - \epsilon)] dF^n(\epsilon)}. \end{aligned}$$

Clearly, the distribution $F_{(n)}(x | \epsilon_{(n)} > \bar{p} - v)$ is increasing in n in the sense of first-order stochastic dominance (FOSD). This fact, together with log-concavity of g (which implies that $\frac{g(\bar{p}-x)}{1-G(\bar{p}-x)}$ is decreasing in x), ensures that (A.3) is decreasing in n as required. Hence, $\frac{d\bar{p}}{dn} \geq 0$. When $n \rightarrow \infty$ and $\epsilon < \infty$, the distribution of F^n collapses to a single point $\bar{\epsilon}$, so $\bar{p} = \arg \max_p \{p(1 - G(p - \bar{\epsilon}))\}$, the solution of which is exactly τ^m .

Recall the equilibrium price without steering is

$$p^* = \arg \max_p \left\{ p \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} [1 - F(\max\{\epsilon - p^*, -v\} + p)] dF(\epsilon)^{n-1} dG(v) \right\},$$

and it is the highest when $n = 1$, in which case $p_{n=1}^* = \arg \max_p \left\{ p \int_{\underline{\epsilon}}^{\bar{\epsilon}} [(1 - G(p - \epsilon)) f(\epsilon)] d\epsilon \right\} = \bar{p}_{n=1}$. We know \bar{p} is increasing in n , and so $\bar{p} \geq p^*$ for all $n \geq 1$, as required. We already know the equilibrium commission equals $\bar{p} > 0$ when M steers with commitment, which is obviously higher than zero commission. Since price is higher with steering, it is immediately that CS , $\sum \pi_i$, and W are lower than the equilibrium without steering. Finally, the result of $\lim_{n \rightarrow \infty} \bar{p} \rightarrow \tau^m$ when M steers immediately implies Proposition 5 still holds. ■

In practice it would be difficult for M to credibly commit to recommending the highest commission product, given that such a recommendation is not sequentially rational. One possible way to implement this would be via an auction mechanism in which all firms could bid openly (so they could see each others' bids, thereby ensuring M sticks to its announced recommendation rule). Among other implementation issues, such a mechanism may be susceptible to collusion between the competing firms. Moreover, including a cap on prices in the mechanism may raise vertical price-fixing issues. Nonetheless, Proposition 9 remains a useful theoretical benchmark showing the highest possible profit that M can achieve when it has commitment power over recommendation rules, and that our main results continue to hold true in this case.

A.5 Equilibrium selection

In the main text, we claim that if we focus on the class of informative (i.e. non-blabbing) equilibria where the first-ranked product has a strictly higher probability to be inspected by consumers relative to other lower-ranked or unranked products, then the informative equilibrium with steering characterized in Section 3.1 is the unique symmetric equilibrium outcome — in particular, consumers inspect the top-ranked item first.

To prove this, suppose by contradiction there exists another candidate symmetric equilibrium in which the top-ranked item has a strictly higher probability to be inspected but consumers do not inspect the top-ranked item first. Therefore, consumers either inspect some lower-ranked or unranked products first, or inspect some randomly chosen products. Regardless of what consumers do, notice that in this candidate equilibrium M necessarily ranks the most suitable item in the first rank. This is because doing so yields the highest ex-ante probability of consumers purchasing something, given that by assumption the top-ranked item has a strictly higher probability to be inspected. Expecting this, consumers must choose to inspect the top-ranked item first, contradicting the initial supposition. Thus, we conclude in any symmetric equilibria where the first-ranked product has a strictly higher probability to be inspected, consumers must inspect the first-ranked product first, and the unique outcome is established by the analysis in Section 3.1.

A.6 Quasi-concavity of profit function

To prove quasi-concavity of the profit function, we first show that firm i 's demand function (2) is globally log-concave in (p_i, τ_i) when f is log-concave and G is linear (i.e., g is constant). Recall that firm i 's demand equals the following probability:

$$\Pr \left(\epsilon_i - p_i \geq \max \left\{ -G^{-1} \left(1 - \frac{\tau_i^*}{\tau_i} (1 - G(p^* - \hat{\epsilon})) \right), -v \right\} \right),$$

which can be rewritten as

$$\begin{aligned} D_i &= \Pr \left(p^* - G^{-1} \left(1 - \frac{\tau_i}{\tau^*} (1 - G(p_i - \epsilon_i)) \right) > \hat{\epsilon} \text{ and } \epsilon_i - p_i > -v \right) \\ &= \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\Pr \left(p^* - G^{-1} \left(1 - \frac{\tau_i}{\tau^*} (1 - G(p_i - \epsilon_i)) \right) > \hat{\epsilon} \right) (1 - G(p_i - \epsilon_i)) \right] d\epsilon_i, \\ &= \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[F \left(p^* - G^{-1} \left(1 - \frac{\tau_i}{\tau^*} (1 - G(p_i - \epsilon_i)) \right) \right)^{n-1} (1 - G(p_i - \epsilon_i)) f(\epsilon_i) \right] d\epsilon_i, \end{aligned} \quad (\text{A.4})$$

where the second equality is due to the conditional independence of the two events after conditioning on ϵ_i . The key step of our proof is to show that the integrand function in (A.4):

$$I(p_i, \tau_i, \epsilon_i) \equiv F \left(p^* - G^{-1} \left(1 - \frac{\tau_i}{\tau^*} (1 - G(p_i - \epsilon_i)) \right) \right)^{n-1} (1 - G(p_i - \epsilon_i)) f(\epsilon_i)$$

is log-concave for $(p_i, \tau_i, \epsilon_i) \in [\underline{v} + \underline{\epsilon}, v + \bar{\epsilon}]^2 \times [\underline{\epsilon}, \bar{\epsilon}]$.

We first claim that $I(p_i, \tau_i, \epsilon_i)$ is log-concave within the convex set

$$S \equiv \left\{ (p_i, \tau_i, \epsilon_i) \in [\underline{v} + \underline{\epsilon}, v + \bar{\epsilon}]^2 \times [\underline{\epsilon}, \bar{\epsilon}] \mid \epsilon_i - p_i \geq -\bar{v} \right\}.$$

By assumption $f(\epsilon_i)$ and $(1 - G(p_i - \epsilon_i))$ are log-concave for $(p_i, \tau_i, \epsilon_i) \in S$. Given that log-concavity is preserved by multiplication, to establish log-concavity of $I(p_i, \tau_i, \epsilon_i)$, it remains to verify the log-concavity of $F(p^* - G^{-1}(1 - \frac{\tau_i}{\tau^*}(1 - G(p_i - \epsilon_i))))^{n-1}$ in set S . The latter, however, is simply a $n - 1$ times self multiplication of $F(p^* - G^{-1}(1 - \frac{\tau_i}{\tau^*}(1 - G(p_i - \epsilon_i))))$, so it remains to check the following claim:

Claim 5 *If f is log-concave and G is linear, then $F(p^* - G^{-1}(1 - \frac{\tau_i}{\tau^*}(1 - G(p_i - \epsilon_i))))$ is log-concave for $(p_i, \tau_i, \epsilon_i) \in S$.*

Proof. Consider first the case $\epsilon_i - p_i \geq 0$. Before proceeding, we first denote $\eta \equiv \frac{d}{dx} \left(\frac{f(x)}{F(x)} \right) < 0$, which is negative because log-concavity of f implies decreasing reverse hazard rate $\frac{f}{F}$. Also, denote

$$\psi \equiv -G^{-1} \left(1 - \frac{\tau_i}{\tau^*} (1 - G(p_i - \epsilon_i)) \right) = \frac{\tau_i}{\tau^*} (\bar{v} + \epsilon_i - p_i) - \bar{v},$$

where the second equality is due to the linearity assumption of G . By implicit differentiation,

$$\frac{\partial \psi}{\partial p_i} = -\frac{\partial \psi}{\partial \epsilon_i} = -\frac{\tau_i}{\tau^*}; \text{ and } \frac{\partial \psi}{\partial \tau_i} = \frac{\bar{v} + \epsilon_i - p_i}{\tau^*}.$$

It is straightforward to see that all relevant second-order derivatives and cross-derivatives are zero, except $\frac{\partial^2 \psi}{\partial \tau_i \partial p_i} = -\frac{\partial^2 \psi}{\partial \tau_i \partial \epsilon_i} = -\frac{1}{\tau^*} < 0$, and likewise for $\frac{\partial^2 \psi}{\partial p_i \partial \tau_i} = \frac{\partial^2 \psi}{\partial \epsilon_i \partial \tau_i} = -\frac{1}{\tau^*}$ due to symmetry.

Our objective is to show that $\ln(F(\psi + p^*))$ is concave $(p_i, \tau_i, \epsilon_i)$. The first-order derivatives are:

$$\begin{aligned} \frac{\partial \ln(F(\psi + p^*))}{\partial p_i} &= \frac{f(\psi + p^*)}{F(\psi + p^*)} \frac{\partial \psi}{\partial p_i} \\ \frac{\partial \ln(F(\psi + p^*))}{\partial \epsilon_i} &= \frac{f(\psi + p^*)}{F(\psi + p^*)} \frac{\partial \psi}{\partial \epsilon_i} \\ \frac{\partial \ln(F(\psi + p^*))}{\partial \tau_i} &= \frac{f(\psi + p^*)}{F(\psi + p^*)} \frac{\partial \psi}{\partial \tau_i}. \end{aligned}$$

The corresponding Hessian matrix is

$$\begin{aligned} H &= \begin{pmatrix} \frac{\partial^2 \ln(F(\psi + p^*))}{\partial p_i^2} & \frac{\partial^2 \ln(F(\psi + p^*))}{\partial p_i \partial \epsilon_i} & \frac{\partial^2 \ln(F(\psi + p^*))}{\partial p_i \partial \tau_i} \\ \frac{\partial^2 \ln(F(\psi + p^*))}{\partial \epsilon_i \partial p_i} & \frac{\partial^2 \ln(F(\psi + p^*))}{\partial \epsilon_i^2} & \frac{\partial^2 \ln(F(\psi + p^*))}{\partial \epsilon_i \partial \tau_i} \\ \frac{\partial^2 \ln(F(\psi + p^*))}{\partial \tau_i \partial p_i} & \frac{\partial^2 \ln(F(\psi + p^*))}{\partial \tau_i \partial \epsilon_i} & \frac{\partial^2 \ln(F(\psi + p^*))}{\partial \tau_i^2} \end{pmatrix} \\ &= \begin{pmatrix} \left(\frac{\tau_i}{\tau^*}\right)^2 \eta & -\left(\frac{\tau_i}{\tau^*}\right)^2 \eta & -\left(\frac{\tau_i}{\tau^*}\right) \left(\frac{\bar{v} + \epsilon_i - p_i}{\tau^*}\right) \eta - \frac{1}{\tau^*} \frac{f(\psi + p^*)}{F(\psi + p^*)} \\ -\left(\frac{\tau_i}{\tau^*}\right)^2 \eta & \left(\frac{\tau_i}{\tau^*}\right)^2 \eta & \left(\frac{\tau_i}{\tau^*}\right) \left(\frac{\bar{v} + \epsilon_i - p_i}{\tau^*}\right) \eta + \frac{1}{\tau^*} \frac{f(\psi + p^*)}{F(\psi + p^*)} \\ -\left(\frac{\tau_i}{\tau^*}\right) \left(\frac{\bar{v} + \epsilon_i - p_i}{\tau^*}\right) \eta - \frac{1}{\tau^*} \frac{f(\psi + p^*)}{F(\psi + p^*)} & \left(\frac{\tau_i}{\tau^*}\right) \left(\frac{\bar{v} + \epsilon_i - p_i}{\tau^*}\right) \eta + \frac{1}{\tau^*} \frac{f(\psi + p^*)}{F(\psi + p^*)} & \left(\frac{\bar{v} + \epsilon_i - p_i}{\tau^*}\right)^2 \eta \end{pmatrix}, \end{aligned}$$

where all η are evaluated at $\psi + p^*$. To show H is negative semi-definite, we check that the determinants of its leading principal minors $H1$, $H2$ and $H3$ alternate in sign as follows:

$$\det(H1) = \left(\frac{\tau_i}{\tau^*}\right)^2 \eta < 0;$$

$$\det(H2) = \det \begin{bmatrix} \left(\frac{\tau_i}{\tau^*}\right)^2 \eta & -\left(\frac{\tau_i}{\tau^*}\right)^2 \eta \\ -\left(\frac{\tau_i}{\tau^*}\right)^2 \eta & \left(\frac{\tau_i}{\tau^*}\right)^2 \eta \end{bmatrix} \geq 0;$$

As for $\det(H3) = \det(H)$, to simplify the notation, denote H_{ij} as the (i, j) entry of the Hessian matrix.

Then

$$\begin{aligned}
\det(H3) &= H_{11}H_{22}H_{33} + H_{12}H_{23}H_{31} + H_{21}H_{32}H_{13} \\
&\quad - (H_{31}H_{22}H_{13} + H_{11}H_{32}H_{23} + H_{33}H_{12}H_{21}) \\
&= \eta \left(\frac{\tau_i}{\tau^*}\right)^2 \left[\eta^2 \left(\frac{\bar{v} + \epsilon_i - p_i}{\tau^*}\right)^2 \left(\frac{\tau_i}{\tau^*}\right)^2 - 2H_{23}H_{13} \right] \\
&\quad - \eta \left(\frac{\tau_i}{\tau^*}\right)^2 \left[\eta \left(\frac{\bar{v} + \epsilon_i - p_i}{\tau^*}\right)^2 \eta \left(\frac{\tau_i}{\tau^*}\right)^2 + H_{13}^2 + H_{23}^2 \right],
\end{aligned}$$

where we have invoked symmetry of H (i.e., $H_{23} = H_{32}$, $H_{31} = H_{13}$) in the second equality. Rearranging,

$$\begin{aligned}
\det(H3) &= -\eta \left(\frac{\tau_i}{\tau^*}\right)^2 [2H_{23}H_{31} + H_{13}^2 + H_{23}^2] \\
&= -\eta \left(\frac{\tau_i}{\tau^*}\right)^2 [-2H_{13}^2 + H_{13}^2 + H_{13}^2] \leq 0,
\end{aligned}$$

where the second equality is due to $H_{23} = -H_{13}$. Therefore, H is indeed negative semi-definite. ■

Next, it is easy to see that $I(p_i, \tau_i, \epsilon_i) = 0$ for $(p_i, \tau_i, \epsilon_i) \notin S$, which follows directly from the fact that $1 - G(p_i - \epsilon_i) = 0$ whenever $\epsilon_i - p_i < -\bar{v}$. Combining this observation with the analysis from the previous paragraph, we have established that $I(p_i, \tau_i, \epsilon_i)$ is a function that is log-concave in convex set S and equals to zero elsewhere. By Prékopa (1971), we thus know that $I(p_i, \tau_i, \epsilon_i)$ is log-concave in the entire space $[\underline{v} + \underline{\epsilon}, v + \bar{\epsilon}] \times [\underline{\epsilon}, \bar{\epsilon}]$, as required. We now invoke the following result by Prékopa (1973), which shows that integration preserves log-concavity:

Lemma 1 (Prékopa, 1973) *Let $f(x, y)$ be a function of $n + m$ variables where x is an n -component and y is an m -component vector. Suppose that f is log-concave in \mathbb{R}^{n+m} and let A be a convex subset of \mathbb{R}^m . Then, the function of the variable x*

$$\int_A f(x, y) dy$$

is log-concave in the entire space \mathbb{R}^n .

Given that $I(p_i, \tau_i, \epsilon_i)$ is log-concave for $(p_i, \tau_i, \epsilon_i) \in [\underline{v} + \underline{\epsilon}, v + \bar{\epsilon}] \times [\underline{\epsilon}, \bar{\epsilon}]$ and we are integrating over ϵ_i in the fixed convex set $[\underline{\epsilon}, \bar{\epsilon}]$, the lemma above by Prékopa implies that the demand function D_i in (A.4) is log-concave in (p_i, τ_i) . It follows that $\Pi_i = (p - \tau_i) D_i$ is log-concave hence quasi-concave.

When G is non-linear, an analytical proof for quasiconcavity is difficult because $\bar{x}(\epsilon)$ is a composite function of G^{-1} and G , which makes the evaluation of the Hessian matrix intractable. In order to determine the global of the profit function in this case, we rely on numerical calculations. Specifically, we plot firm i 's profit function in equilibrium when F and G follows $N(\mu, \sigma)$, for all combinations of $\mu \in \{0, 1, 2\}$, $\sigma \in \{1, 2, 3\}$, and $n \in \{2, 3, 5, 10\}$. Details and codes of the numerical calculations are available from the authors upon request. For example, for $\mu = 0$ and $\sigma = 1$, we obtain the set of plots in Figure 2. In all the cases considered, we confirmed from the contour plots that the quasiconcavity assumption was satisfied, suggesting it does not require very special conditions to hold.

A.7 Lump-sum payments

In establishing the informative equilibrium with steering in Proposition 1, we have ruled out the feasibility of a firm offering M a lump-sum payment in return for M steering all consumers to that firm given that it is not sequentially rationale for M to do so. In this section, we examine what happens when we relax the requirement of sequential rationality such that a lump-sum contract becomes feasible.

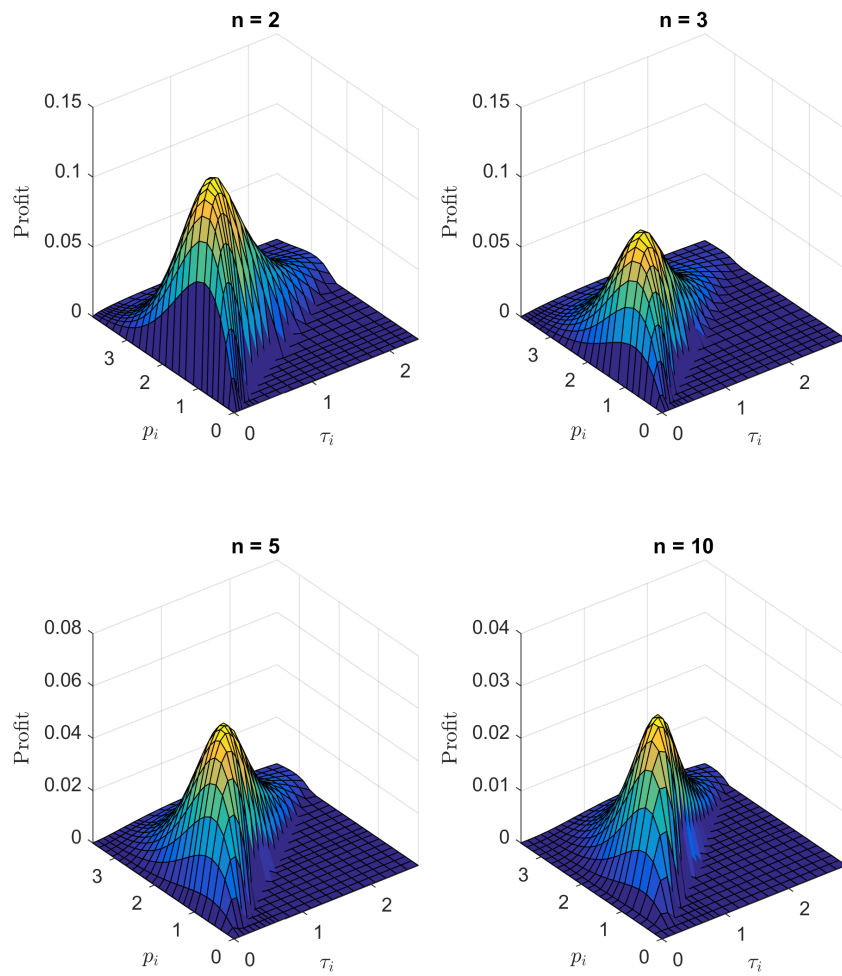


Figure 2: Profit function $\Pi(p_i, \tau_i)$, assuming all other firms set the equilibrium price and commission.

Obviously, there is no equilibrium in which any firm offers a lump-sum contract to M in equilibrium in return for being exclusively recommended. Whenever such a contract is offered in equilibrium, consumers rationally expect that M 's recommendation is completely biased and uninformative, so that they will search through products as if there is no intermediation.

In what follows, we examine whether Proposition 1 remains a valid equilibrium when firms can deviate off-equilibrium by offering lump-sum contracts. Suppose that lump-sum contracts are unobservable to consumers so that their search behavior remains the same as in Section 3.1. For M to agree to an exclusive lump-sum contract, a deviating firm (say i) must offer a lump-sum payment T that compensates M for the total commission that M obtains from all other firms, i.e.,

$$T = \tau^* \int_{\underline{v}}^{\bar{v}} [1 - F(p^* - v)^n] dG(v). \quad (\text{A.5})$$

Provided that M agrees to the contract, firm i no longer needs to pay any commission to M , and the firm becomes a monopoly with net deviation profit

$$\Pi'_i - T = \max_p \left\{ p \int_{\underline{v}}^{\bar{v}} [1 - F(p - v)] dG(v) \right\} - T.$$

Given that firm i is earning a non-negative profit in the initial equilibrium, a necessary condition for the deviation by firm i to be profitable is $\Pi'_i - T > 0$. Note from (A.5) that T is a function of n . Suppose $\bar{\epsilon}$ is finite and n is sufficiently large, Proposition 4 implies that M earns a “ $\bar{\epsilon}$ -product monopoly profit”, that is

$$\begin{aligned} & \Pi'_i - T \\ \rightarrow & \Pi'_i - \max \{ p(1 - G(p - \bar{\epsilon})) \} < 0. \end{aligned}$$

It follows that when n is sufficiently large, the deviating exclusive contract is not profitable for firm i .

In cases where n is small, in principle the deviation by lump-sum contract may be profitable so that the informative equilibrium in Proposition 1 becomes unsustainable when such contracts can be used. When that happens, the equilibrium becomes an uninformative one whereby all consumers ignore M 's recommendation and search sequentially through the products as if there is no intermediary, and all firms pay no commission to M . Hence, allowing for exclusive lump-sum contracts and M 's commitment to such contracts may sometimes cause the informative equilibrium to break down.²

A.8 Additional figures for Section 4

Figure 3 compares the price, consumer surplus, and welfare when M steers versus when it does not, whereby F and G are linear with distribution support $[-1, 1]$.

Figure 4 corresponds to Figure 1 in Section 4.3 of the main text, assuming F and G take the standard normal distribution $N(0, 1)$. The equilibrium price and commission still increase with n , even though condition (8) does not hold.

A.9 Derivation of Section 4.4

In this section, we derive consumers' optimal search rule and the resulting demand function for each firm in the model of Section 4.4 in the main text. Suppose, for the moment, consumers learn the realized value of v before any search. It is then well known from Kohn and Shavell (1974) that consumers' optimal search rule

²Nonetheless, we have confirmed that when both F and G are linear with distribution support $[-1, 1]$, the net deviation profit is strictly smaller than the equilibrium profit for firm i for all $n \geq 2$ so that the information equilibrium still holds in this case.

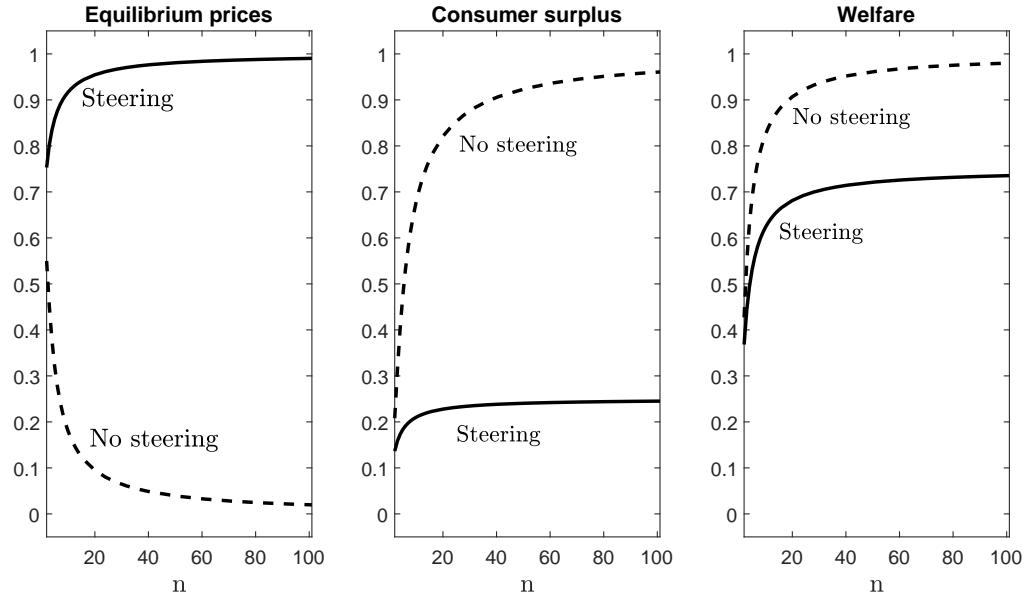


Figure 3: Price, consumer surplus and welfare when M steers versus when it does not, assuming F and G are $U[-1, 1]$.

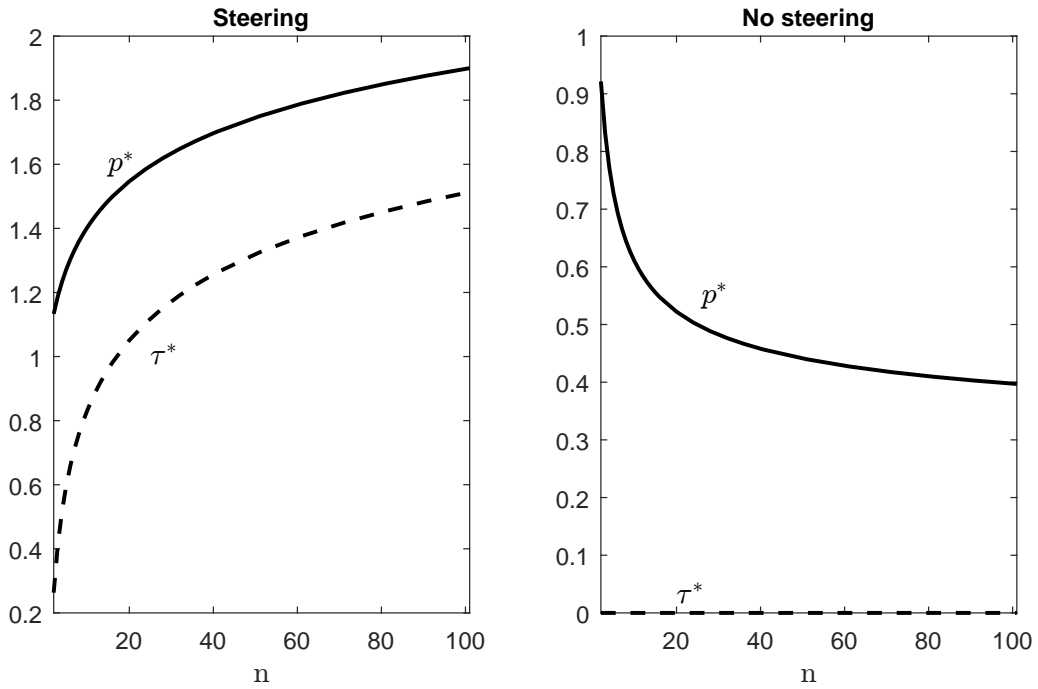


Figure 4: Prices and commissions when M steers versus when it does not, assuming F and G are $N(0, 1)$.

in this environment is stationary and described by the standard cutoff rule. Define the reservation value \tilde{x} as the solution to

$$\int_{\tilde{x}}^{\bar{\epsilon}} [\epsilon - \tilde{x}] dF(\epsilon) = s.$$

The left-hand side denotes the expected incremental benefit from one more search given the offer in hand \tilde{x} , while the right-hand side is the incremental search cost. Given that v is constant for any particular consumer, each consumer employs the following cutoff strategy when searching: (i) stops searching further if $\max\{v + \epsilon_i - p_i, 0\} \geq v + \tilde{x} - \tilde{p}$, or (ii) continues to search the next firm otherwise. Following the standard results, $v + \tilde{x} - \tilde{p}$ also represents consumers' expected surplus from initiating search once v is known.

Now suppose, consistent with our baseline model, that consumers do not actually know v before searching. Then they will carry out the first search as long as the ex-ante net surplus is positive. After the first search, consumers fully learn the realized value of v and the subsequent search problem of consumers is exactly described by the previous paragraph. Consumers with $v + \tilde{x} - \tilde{p} < 0$ expect no surplus gain from searching further relative to the outside option, so that they will stop searching and either purchase the first product or the outside option. On the other hand, consumers with $v + \tilde{x} - \tilde{p} \geq 0$ expect a positive surplus gain from costly search and they will continue searching until they find an option which gives them a surplus of at least $v + \tilde{x} - \tilde{p}$.

From the consumer search rule above, the derivation of demand facing firms is straightforward. For consumers with $v \geq \tilde{p} - \tilde{x}$, a deviating firm i 's conditional demand follows the standard search model and it is given by

$$(1 - F(\tilde{x} - \tilde{p} + p_i)) \sum_{k=0}^{\infty} F(\tilde{x})^k = \frac{1 - F(\tilde{x} - \tilde{p} + p_i)}{1 - F(\tilde{x})}. \quad (\text{A.6})$$

On the other hand, for consumers with $v < \tilde{p} - \tilde{x}$, firm i effectively becomes a local monopoly over these consumers since they do not search further. Firm i 's conditional demand in this case is

$$1 - F(p_i - v). \quad (\text{A.7})$$

Integrating both conditional demands in (A.6) and (A.7) over v gives the demand function

$$D_i(p_i) = \int_{\min\{\tilde{p}-\tilde{x}, \bar{v}\}}^{\bar{v}} \left[\frac{1 - F(\tilde{x} - \tilde{p} + p_i)}{1 - F(\tilde{x})} \right] dG(v) + \int_{\underline{v}}^{\min\{\tilde{p}-\tilde{x}, \bar{v}\}} [1 - F(p_i - v)] dG(v)$$

as stated in the proof of Proposition 5 in the appendix of the main text. The equilibrium characterization follows from Proposition 5 in the main text.

B Policy implications

In this section of the online appendix, we analyze in detail the omitted analysis described in Section 5 of the main text.

B.1 Concern for suitability

We extend our baseline model by allowing M to have a direct concern for consumer surplus. To microfound this possibility in the simplest possible fashion, we assume that, following firm i being M 's recommended firm (or top ranked firm), the consumer lodges a complaint for an inappropriate recommendation with probability ρ , which results in a fixed penalty of α for M . We assume that ρ is an affine function of $1 - G(p_i - \epsilon_i)$:

$$\rho = \rho_i = \beta_1 - \beta_2 (1 - G(p_i - \epsilon_i)),$$

where $\beta_1, \beta_2 \geq 0$ are such that $0 \leq \rho \leq 1$. This functional form means that ρ is decreasing in the surplus that the consumer obtains from the product, so recommending a product that offers less surplus is more likely to lead to a consumer complaint.

In this environment, we can construct the same informative equilibrium as in Proposition 1 whereby consumers only search once and M gives unbiased recommendations in the equilibrium. The only exception is that M 's recommendation off-equilibrium path is now based on the following decision rule: the top ranked product i by M satisfies

$$\tau_i (1 - G(p_i - \epsilon_i)) - \alpha \rho_i \geq \max_{j \neq i} \{ \tau^* (1 - G(p^* - \epsilon_j)) \} - \alpha \rho_j,$$

or equivalently, after cancelling out common terms:

$$(\tau_i + \beta_2 \alpha) (1 - G(p_i - \epsilon_i)) \geq \max_{j \neq i} \{ (\tau^* + \beta_2 \alpha) (1 - G(p^* - \epsilon_j)) \}. \quad (\text{B.1})$$

Therefore, a higher penalty α implies a greater weight that M assigns to consumer surplus, meaning that its recommendation is less affected by commission differences across firms. Broadly interpreted, the parameter α captures M 's concern for product suitability, as in Inderst and Ottaviani (2012a).

Given that all other aspects of the model are similar to the baseline model, we can derive the equilibrium prices and fees using the usual first order conditions. Provided that the firms' profit function is globally quasi-concave, we can state the equilibrium price and commissions with the following two equations, which parallel expressions (5) and (7) in the main text:

$$p^* = \tau^* + \frac{\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} [1 - F(\max\{\epsilon, p^* - v\})] dF(\epsilon)^{n-1} dG(v)}{\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} f(\max\{\epsilon, p^* - v\}) dF(\epsilon)^{n-1} dG(v)},$$

and

$$\tau^* = \max \left\{ 0, \frac{\int_{\underline{v}}^{\bar{v}} \int_{p^*-v}^{\bar{\epsilon}} \left[\frac{1-G(p^*-\epsilon)}{g(p^*-\epsilon)} f(\epsilon) \right] dF(\epsilon)^{n-1} dG(v)}{\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} f(\max\{\epsilon, p^* - v\}) dF(\epsilon)^{n-1} dG(v)} - \beta_2 \alpha \right\}.$$

We then have the following result:

Proposition 11 *If the informative equilibrium exists, then:*

1. *The equilibrium price and commission levels decrease with M 's concern for product suitability (α).*
2. *Consumer surplus, firms' profit, and welfare increase with M 's concern for product suitability (α).*

Proof. A total differentiation on the system of equation (p^*, τ^*) and a direct application of Cramer's rule, similar to that in the proof of Proposition 13, yields the proposition. ■

B.2 Informed consumers

We extend our baseline model by allowing for two types of consumers. With probability λ , a consumer is informed. Such a consumer knows the prices and the realizations of all match utilities before making her purchase decision. Equivalently, she has zero (search) costs of inspecting products, and so will always inspect every product. With the remaining probability $1 - \lambda$, a consumer is uninformed and behaves exactly the same as the consumers in our baseline model. All purchases still go through M , meaning it receives commissions for purchases by both the informed and uninformed consumers. The realization of a consumer's type is not known to firms and M . The analysis below does not depend on whether consumers observe the decomposition of v_i after inspection.

In this setup, the model parameter λ is designed to capture that consumers are sometimes informed and do not fully rely on M 's recommendation. Hence, one can interpret λ as the extent to which a representative consumer is informed. In what follows, we interpret an increase in λ as the representative consumer becoming more informed, and we explore how an increase in λ affects the equilibrium commission and price.

We focus on the informative equilibrium with steering, as in the baseline model. In particular: (i) all firms adopt the same strategy; (ii) M ranks all products in order of expected commission; and (iii) consumers (if uninformed) inspect the highest ranked product without searching further given they believe that the highest-ranked product gives them the highest surplus.

Demand from uninformed consumers is given by (2), which we denote as

$$D_i^U = \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} [1 - F(\max\{\bar{x}_i(\epsilon), -v\} + p_i)] dF(\epsilon)^{n-1} dG(v).$$

Meanwhile, an informed consumer purchases a product i if $v + \epsilon_i - p_i \geq \max_{j \neq i} \{v + \epsilon_j - p_j, 0\}$. Provided that all firms set the equilibrium price at p^* , a deviating firm i 's demand from informed consumers can be derived as

$$D_i^I = \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} [1 - F(\max\{\epsilon - p^*, -v\} + p_i)] dF(\epsilon)^{n-1} dG(v).$$

Importantly, note that D_i^I is independent of commissions, as opposed to D_i^U , which reflects that an informed consumer cannot be steered by M . Then, firm i 's total demand is the weighted sum of the two demand components, i.e., $\lambda D_i^I + (1 - \lambda) D_i^U$. A typical deviating firm i solves

$$\max_{p_i, \tau_i} \Pi_i = \max_{p_i, \tau_i} (p_i - \tau_i) [\lambda D_i^I + (1 - \lambda) D_i^U]. \quad (\text{B.2})$$

The demand derivatives and first-order conditions can be obtained through similar steps to those used to prove Proposition 1, with the only new step being that

$$\frac{\partial D_i^I}{\partial p_i} = - \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} f(\max\{\epsilon - p^*, -v\} + p_i) dF(\epsilon)^{n-1} dG(v) < 0.$$

A useful observation is that, after imposing $p_i = p^*$ and $\tau_i = \tau^*$, the equilibrium demand and demand derivatives of informed and uninformed consumers coincide exactly:

$$\begin{aligned} \frac{\partial D_i^I}{\partial p_i} &= \frac{\partial D_i^U}{\partial p_i} = - \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} f(\max\{\epsilon, p^* - v\}) dF(\epsilon)^{n-1} dG(v) \\ D_i^I &= D_i^U = \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} [1 - F(\max\{\epsilon, p^* - v\})] dF(\epsilon)^{n-1} dG(v). \end{aligned}$$

Consequently, we can state the equilibrium price and commissions with the following two equations, which parallel the expressions (5) and (7) in the main text:

$$p^* = \tau^* + \frac{\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} [1 - F(\max\{\epsilon, p^* - v\})] dF(\epsilon)^{n-1} dG(v)}{\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} f(\max\{\epsilon, p^* - v\}) dF(\epsilon)^{n-1} dG(v)}, \quad (\text{B.3})$$

and

$$\tau^* = (1 - \lambda) \frac{\int_{\underline{v}}^{\bar{v}} \int_{p^* - v}^{\bar{\epsilon}} \left[\frac{1 - G(p^* - \epsilon)}{g(p^* - \epsilon)} f(\epsilon) \right] dF(\epsilon)^{n-1} dG(v)}{\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} f(\max\{\epsilon, p^* - v\}) dF(\epsilon)^{n-1} dG(v)}. \quad (\text{B.4})$$

Other aspects of the equilibrium characterization remain the same as the baseline model. Formally:

Proposition 12 (*Informative equilibrium*) *If profit function (B.2) is globally quasiconcave in (p_i, τ_i) , then the informative equilibrium exists in which:*

1. All firms set p^* and τ^* given by (B.3) and (B.4);
2. M recommends the product with highest expected commission; and
3. All consumers inspect the recommended product without searching further.

Note that equilibrium existence is non-trivial because firms may profitably deviate from the informative equilibrium by offering no commission and instead focusing on selling to the informed consumers. One sufficient condition for equilibrium existence is to have λ sufficiently close to zero, so that the aforementioned deviation is unprofitable, and the sufficiency condition (linearity of G) employed in the baseline model becomes directly applicable to establish quasiconcavity of the profit function (B.2). If F and G are linear with distribution support $[-1, 1]$, we have numerically verified that the informative equilibrium is sustainable even at moderate λ provided that n is not too large. For example, it is sustainable for $\lambda \leq 0.5$ provided $n \leq 4$, and for $\lambda \leq 0.1$ provided $n \leq 10$.

We now show how the equilibrium outcome changes with λ . To proceed, we need to define consumer surplus, firms' profit, and welfare in this context. Recall that M recommends the most suitable product for uninformed consumers, so that the equilibrium consumer surplus for both informed and uninformed consumers coincides. Since only uninformed consumers incur search cost and they incur it only once, consumer surplus can be written as

$$CS \equiv \int_{\underline{v}}^{\bar{v}} \int_{v+p^*}^{\bar{\epsilon}} [v + \epsilon - p^*] dF(\epsilon)^n dG(v) - (1 - \lambda) s,$$

which is same as the consumer surplus expression in Section 4 of the main text. Likewise,

$$\sum \pi_i = (p^* - \tau^*) \int_{\underline{v}}^{\bar{v}} [1 - F(p^* - v)^n] dG(v)$$

$$W = \int_{\underline{v}}^{\bar{v}} \int_{p^*-v}^{\bar{\epsilon}} [\epsilon + v] dF(\epsilon)^n dG(v) - (1 - \lambda) s.$$

Proposition 13 *Consider the model with informed and uninformed consumers. If the informative equilibrium exists, then:*

1. The equilibrium price and commission levels decrease with the probability of consumers being informed (λ).
2. Consumer surplus, firms' profit, and welfare increase with the probability of consumers being informed (λ).

Proof. Consider the first part of the proposition: p^* and τ^* are increasing in λ . As in the proof of Proposition 1, denote

$$\phi_1 \equiv \frac{\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} [1 - F(\max\{\epsilon, p^* - v\})] dF(\epsilon)^{n-1} dG(v)}{\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} f(\max\{\epsilon, p^* - v\}) dF(\epsilon)^{n-1} dG(v)}$$

and

$$\phi_2 \equiv \frac{\int_{\underline{v}}^{\bar{v}} \int_{p^*-v}^{\bar{\epsilon}} \left[\frac{1-G(p^*-\epsilon)}{g(p^*-\epsilon)} f(\epsilon) \right] dF(\epsilon)^{n-1} dG(v)}{\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} f(\max\{\epsilon, p^* - v\}) dF(\epsilon)^{n-1} dG(v)}.$$

In the last part of the proof of Proposition 1, we showed that $d\phi_1/dp^* < 0$ and $d\phi_2/dp^* < 0$. Total differentiation of (B.3) and (B.4), in matrix form, gives

$$\begin{bmatrix} 1 - \frac{d\phi_1}{dp^*} & -1 \\ -(1 - \lambda) \frac{d\phi_2}{dp^*} & 1 \end{bmatrix} \begin{bmatrix} \frac{dp^*}{d\lambda} \\ \frac{d\tau^*}{d\lambda} \end{bmatrix} = \begin{bmatrix} 0 \\ -\phi_2 \end{bmatrix}.$$

Denote

$$Det \equiv \det \begin{pmatrix} 1 - \frac{d\phi_1}{dp^*} & -1 \\ -(1-\lambda) \frac{d\phi_2}{dp^*} & 1 \end{pmatrix} = 1 - \frac{d\phi_1}{dp^*} - (1-\lambda) \frac{d\phi_2}{dp^*} > 0.$$

By Cramer's rule,

$$\frac{dp}{d\lambda} = \frac{1}{Det} \begin{vmatrix} 0 & -1 \\ -\phi_2 & 1 \end{vmatrix} = \frac{\phi_2}{Det} < 0 \quad \text{and} \quad \frac{d\tau}{d\lambda} = \frac{1}{Det} \begin{vmatrix} 1 - \frac{d\phi_1}{dp^*} & 0 \\ -(1-\lambda) \frac{d\phi_2}{dp^*} & -\phi_2 \end{vmatrix} = -\frac{\phi_2}{Det} \left(1 - \frac{d\phi_1}{dp^*}\right) < 0,$$

as required. It is useful to note that $|d\tau/d\lambda| > |dp^*/d\lambda|$, which signifies an incomplete pass through of the commissions into product prices.

Consider the second part of the proposition. First, CS and W are clearly decreasing in p^* , and p^* is decreasing in λ by the first part of the proposition. Moreover, the search cost incurred is decreasing in λ , so that CS and W indeed increases with λ . As for $\sum \pi_i$, due to incomplete pass through where $|d\tau/d\lambda| > |dp^*/d\lambda|$, firms' equilibrium margin must be increasing with λ while the equilibrium level of sales for each firm is decreasing in p^* (hence increasing in λ). Consequently $\sum \pi_i$ increases with λ . ■

Finally, we show how various results in the baseline model remain robust in this extended model. Formally, we have:

Proposition 14 *Consider the informative equilibrium in Proposition 12:*

1. *Compared to the informative equilibrium with steering, price and commission levels are lower in the equilibrium without steering.*
2. *Compared to the informative equilibrium with steering, Π_M is lower in the equilibrium without steering, while CS , $\sum \pi_i$, and W are higher in the equilibrium without steering.*
3. *If F and G are linear, then τ^* always increases with n . Meanwhile, p^* increases with n if $\lambda < 1/2$, constant in n if $\lambda = 1/2$, and decreases with in n if $\lambda > 1/2$.*

Proof. Parts (1) and (2) follow from the proof of Proposition 13 above by substituting in the special case of $\lambda = 0$. This reflects that the case of $\lambda = 0$ (where all consumers are informed) is mathematically the same as having an equilibrium without steering. It remains to prove part (3). We know from Proposition 12 that p^* and τ^* are respectively pinned down by $p^* = \phi_1 + (1-\lambda)\phi_2$ and $\tau^* = (1-\lambda)\phi_2$. By the implicit function theorem,

$$\frac{dp^*}{dn} = \frac{\partial(\phi_1 + (1-\lambda)\phi_2)/\partial n}{1 - \partial\phi_1/\partial p^* - \partial\phi_2/\partial p^*},$$

so that the sign of dp^*/dn is the same as $\partial(\phi_1 + (1-\lambda)\phi_2)/\partial n$. With the exact same steps as in the proof of Proposition 4 (imposing F is linear), we can show that $d\phi_1/dn \leq 0$, while

$$\begin{aligned} & \phi_1 + (1-\lambda)\phi_2 \\ = & \frac{1 - G(p^* - \bar{\epsilon})}{g(p^* - \bar{\epsilon})} \\ & - \left(\frac{1 - G(p^* - \bar{\epsilon})}{\bar{\epsilon} - \underline{\epsilon}} \right)^{-1} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\left((2\lambda - 1)(1 - G(p^* - \epsilon)) + \frac{(1 - G(p^* - \epsilon))^2}{g(p^* - \epsilon)^2} g'(p^* - \epsilon) \right) \left(\frac{F^{n-1}(\epsilon)}{\bar{\epsilon} - \underline{\epsilon}} \right) \right] d\epsilon, \end{aligned}$$

When G is linear, we have $g' = 0$, so that $\partial(\phi_1 + (1-\lambda)\phi_2)/\partial n$ is negative if $\lambda < 1/2$, zero if $\lambda = 1/2$, and positive if $\lambda > 1/2$.

To show the result on commission, we totally differentiate $\tau^* = \phi_2(p^*)$ to get

$$\frac{1}{1-\lambda} \frac{d\tau^*}{dn} = \frac{\partial\phi_2}{\partial n} + \frac{\partial\phi_2}{\partial p^*} \frac{dp^*}{dn}$$

and it follows from the proof of Proposition 4.2 that $\frac{\partial \phi_2}{\partial n} + \frac{\partial \phi_2}{\partial p^*} \frac{dp^*}{dn} \geq 0$. Therefore, $d\tau^*/dn \geq 0$ regardless of λ . ■

B.3 Mandatory disclosure

We first specify and verify the equilibrium considered. Similar to the benchmark case, consumers hold passive belief over any unobserved prices and commissions (where applicable).

1. **Firms.** All firms set prices and commissions equal to p^* and τ^* respectively;
2. **Intermediary.** For each consumer and at any stage in their search process, M ranks all products in order of expected commission $\tau_i (1 - G(p_i - \epsilon_i))$, with the order of any ties being chosen in favor of the product with the higher surplus $\epsilon_i - p_i$ (and randomly from among any remaining ties);
3. **Unobservant consumers.** Regardless of how many products are ranked, unobservant consumers inspect the highest ranked product (say product i) without searching further, purchasing i if $v + \epsilon_i - p_i \geq 0$, and otherwise purchasing the outside option. They believe that the first-ranked product gives them the highest surplus, and that the surplus of any lower ranked or non-ranked product is (weakly) lower than this. In case M makes no recommendation, these consumers' purchase and search behavior is optimized as if M is absent.
4. **Observant consumers.** If M 's ranking includes at least one of the lowest commission product(s), observant consumers inspect the products sequentially from the highest ranked product (say product i) to the lowest ranked product. If $\tau_i \leq \tau^*$, they stop searching, purchasing i if $v_i - p_i \geq 0$, and otherwise purchasing the outside option. Otherwise, they continue searching until encountering one of the firm(s) that offers the lowest commission, at which point they select a product to buy among the products inspected and the outside option. They believe that M ranks all products in order of expected commission. In case M 's ranking excludes all of the lowest commission product(s), these consumers' purchase and search behavior is optimized as if M is absent.

Proof. In the symmetric equilibrium, M offers an unbiased ranking. Therefore M and consumers' strategy are clearly optimal on the equilibrium path. In what follows, we consider an off-equilibrium path scenario in which a firm i deviates by setting $p_i \neq p^*$ and $\tau_i \neq \tau^*$.

We first check M 's incentives regarding its ranking. Denote

$$j^* \equiv \arg \max_{j \neq i} \{\epsilon_j - p^*\},$$

that is, the highest surplus product excluding i . In the proposed equilibrium, the only decision that matters to M is whether to rank j^* first or to rank i first, because other rankings are either outcome-equivalent or strictly worse than one of these two. Suppose $\tau_i < \tau^*$, so that M 's profit from ranking product i first is $\Pi^M(i) = \tau_i (1 - G(p_i - \epsilon_i))$, while the profit from ranking j^* first is

$$\Pi^M(j^*) = \begin{cases} \lambda \tau^* (1 - G(p^* - \epsilon_{j^*})) + (1 - \lambda) \tau_i (1 - G(p_i - \epsilon_i)) & \text{if } \epsilon_i - p_i > \epsilon_{j^*} - p^* \\ \tau^* (1 - G(p^* - \epsilon_{j^*})) & \text{if } \epsilon_i - p_i \leq \epsilon_{j^*} - p^* \end{cases}.$$

A simple comparison shows $\Pi^M(i) \geq \Pi^M(j^*)$ if and only if $\tau_i (1 - G(p_i - \epsilon_i)) \geq \tau^* (1 - G(p^* - \epsilon_{j^*}))$. Suppose instead $\tau_i > \tau^*$, then M 's profit from ranking product j^* first is $\Pi^M(j^*) = \tau^* (1 - G(p^* - \epsilon_{j^*}))$, while the profit from ranking i first is

$$\Pi^M(i) = \begin{cases} \tau_i (1 - G(p_i - \epsilon_i)) & \text{if } \epsilon_i - p_i > \epsilon_{j^*} - p^* \\ \lambda \tau^* (1 - G(p^* - \epsilon_{j^*})) + (1 - \lambda) \tau_i (1 - G(p_i - \epsilon_i)) & \text{if } \epsilon_i - p_i \leq \epsilon_{j^*} - p^* \end{cases}.$$

Again, $\Pi^M(i) \geq \Pi^M(j^*)$ if and only if $\tau_i(1 - G(p_i - \epsilon_i)) \geq \tau^*(1 - G(p^* - \epsilon_{j^*}))$.

For unobservant consumers, given that they behave exactly the same as the consumers in our baseline model, it follows immediately that the proposed strategy is optimal under the beliefs specified.

For observant consumers, whenever M 's ranking includes at least one of the lowest commission product(s), these consumers have no reason not to inspect the top-ranked product first given their beliefs, because there is no instance in which they can infer that lower-ranked or unranked products are better. Consider an observant consumer who has inspected the top-ranked product, say product i . There are two cases:

- Suppose $\tau_i \leq \tau^*$, i.e., i is one of the lowest commission products. Then the consumer can infer from M 's ranking strategy that $\epsilon_i - p_i \geq \max_{j \neq i} \{\epsilon_j - p_j\}$, and so she has no incentive to keep searching on given the positive search cost. Once the consumer stops searching, she buys either product i or the outside option.
- Suppose instead $\tau_i > \tau^*$. The consumer can infer from M 's ranking strategy that the next product is j^* , and $1 - G(p^* - \epsilon_{j^*}) \leq \frac{\tau_i}{\tau^*} (1 - G(p_i - \epsilon_i))$. After observing v_i and p_i from inspecting product i , the corresponding expected net incremental benefit (or option value) from inspecting the next firm is thus

$$E \left(\max \{v + \epsilon_{j^*} - p^* - \max \{v_i - p_i, 0\}, 0\} | \epsilon_{j^*} - p^* \leq -G^{-1} \left(1 - \frac{\tau_i}{\tau^*} (1 - G(p_i - \epsilon_i)) \right) | v_i \right) - s, \quad (\text{B.5})$$

where the expectation is taken with respect to $\epsilon_i, \epsilon_{j^*}$ and v . Substituting for $\epsilon_i = v_i + v$ and applying iterated expectations, (B.5) becomes

$$E_v \left[E_{\epsilon_{j^*}} \left[\max \{v + \epsilon_{j^*} - p^* - \max \{v_i - p_i, 0\}, 0\} | v + \epsilon_{j^*} - p^* \leq v - G^{-1} \left(1 - \frac{\tau_i}{\tau^*} (1 - G(p_i - \epsilon_i)) \right) \right] | v_i \right] - s.$$

The first component is positive as long as there is a positive mass of v satisfying

$$v_i - p_i < v - G^{-1} \left(1 - \frac{\tau_i}{\tau^*} (1 - G(p_i - \epsilon_i)) \right),$$

which indeed holds given that $\frac{\tau_i}{\tau^*} > 1$ and $v_i \equiv v + \epsilon_i$. Therefore, (B.5) is always positive so that the consumer will inspect the second-ranked product given that search cost is arbitrarily small. After inspecting the second ranked product (that is, product j^* by M 's ranking strategy), the consumer has no incentive to search further. Once she stops, she selects among product i , product j^* , and the outside option to make the purchase.

■

Given the equilibrium characterization above, a deviating firm i 's demand is the same as in the baseline model when $\tau_i \leq \tau^*$:

$$D_i(p_i, \tau_i) = \Pr \left(\epsilon_i - p_i \geq \max_{j \neq i} \{\bar{x}_i(\epsilon_j), v\} \right) \text{ if } \tau_i \leq \tau^*,$$

where $\bar{x}_i(\epsilon) \equiv -G^{-1} \left(1 - \frac{\tau^*}{\tau_i} (1 - G(p^* - \epsilon)) \right)$. If $\tau_i > \tau^*$, unobservant consumers behave the same as in the baseline model. Meanwhile observant consumers buy product i if the following three conditions hold: $\epsilon_i - p_i \geq \max_{j \neq i} \{\bar{x}_i(\epsilon_j)\}$ (i is recommended) and $\epsilon_i - p_i \geq \max_{j \neq i} \{\epsilon_j - p_j, -v\}$ (consumers search beyond the recommended product and still find i to be the best). Given $\frac{\tau^*}{\tau_i} < 1$ implies $\bar{x}_i(\epsilon_j) < \epsilon_j - p_j$, the first condition is non-binding whenever the last condition hold, so that firm i 's demand is simply:

$$D_i(p_i, \tau_i) = \begin{cases} (1 - \lambda) \Pr(\epsilon_i - p_i \geq \max_{j \neq i} \{\bar{x}_i(\epsilon_j), -v\}) \\ \quad + \lambda \Pr(\epsilon_i - p_i \geq \max_{j \neq i} \{\epsilon_j - p_j, -v\}) \end{cases} \text{ if } \tau_i > \tau^*.$$

Clearly, $D_i(p_i, \tau_i)$ is increasing and continuous in τ_i , but not differentiable at $\tau_i = \tau^*$ because $\frac{dD_i}{d\tau_i} |_{\tau_i \rightarrow \tau^* -} = \frac{1}{(1-\lambda)} \frac{dD_i}{d\tau_i} |_{\tau_i \rightarrow \tau^* +}$, That is, the demand derivative $\frac{dD_i}{d\tau_i}$, and by extension the profit derivative $\frac{d\Pi_i}{d\tau_i}$, ‘‘jumps’’

downward at $\tau_i = \tau^*$. This kinked demand form gives rise to the possibility of multiple equilibrium. For example, provided that the relevant second-order conditions hold, then some possible equilibria are

$$\frac{d\Pi_i}{dp_i}\Big|_{p_i=p^*} = \frac{d\Pi_i}{d\tau_i}\Big|_{\tau_i \rightarrow \tau^{*-}} = 0, \quad (\text{B.6})$$

and

$$\frac{d\Pi_i}{dp_i}\Big|_{p_i=p^*} = \frac{d\Pi_i}{d\tau_i}\Big|_{\tau_i \rightarrow \tau^{*+}} = 0, \quad (\text{B.7})$$

as well as other equilibrium that cannot be characterized through first-order conditions (which we do not consider due to tractability issues).

One sufficient condition for the second-order conditions corresponding to (B.6) and (B.7) to hold is to have λ sufficiently close to zero, so that the sufficiency condition (Assumption 1) employed in the baseline model becomes directly applicable to establish quasiconcavity of the profit function in this extension. In the case of F and G are linear with distribution support $[-1, 1]$, we have numerically verified that the informative equilibrium is sustainable even at moderate λ provided that n is not too large. For example, it is sustainable for $\lambda \leq 0.7$ provided $n \leq 5$, and for $\lambda \leq 0.5$ provided $n \leq 20$.

It is easy to see that the equilibrium characterized by (B.6) is the same as the baseline model, in which case commission disclosure has no effect on the equilibrium. The more interesting equilibrium is the one characterized by (B.7), which turns out to be the same equilibrium as the one described by Proposition 12 in the extended model with informed and uninformed consumers. Consequently, the result of Proposition 13 implies that, when $\lambda > 0$, equilibrium price and commission levels are lower, while consumer surplus, firms' profit, and welfare are higher (when compared to the case with $\lambda = 0$). This proves the mandatory disclosure result stated in Proposition 6.³

C Extensions

In this section of the online appendix, we analyze in detail the omitted analysis described in Section 6 of the main text.

C.1 Asymmetric firms

C.1.1 The cost of the second search is high

The derivation of demand in this case is stated in the main text. It remains to prove Proposition 7.

Proof. (Proposition 7). Given that each firm's profit function is the same as in the baseline mode, and that F and G are linear over $[\underline{\epsilon}, \bar{\epsilon}]$ and $[\underline{v}, \bar{v}]$ respectively, we have:

$$\Pi_i = (p_i - c_i - \tau_i) \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[1 - \frac{(\max\{\bar{x}_i(\epsilon), -v\}) + p_i - \underline{\epsilon}}{\bar{\epsilon} - \underline{\epsilon}} \right] \left(\frac{1}{\bar{\epsilon} - \underline{\epsilon}} \right) \left(\frac{1}{\bar{v} - \underline{v}} \right) d\epsilon dv,$$

³We also considered an alternative model where commission payments are observable to all consumers, while consumers face heterogenous search costs randomly distributed over interval $[0, 1]$. In this case, consumers with high search cost do not react to commission changes (as if they are uninformed), while consumers with low search cost search more than once whenever they expect M 's ranking is biased (as if they are observant). Our result that mandatory disclosure reduces price and commission levels remain robust under this alternative model. Details are available from the authors upon request.

where $\bar{x}_i(\epsilon) = \frac{\tau_j}{\tau_i}(\bar{v} + \epsilon - p_j) - \bar{v}$. Then, the first-order conditions can be derived as:

$$\begin{aligned} p_i &= c_i + \tau_i + \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\bar{\epsilon} - \left(\max \left\{ \frac{\tau_j}{\tau_i}(\bar{v} + \epsilon - p_j) - \bar{v}, -v \right\} \right) - p_i \right] d\epsilon dv \\ \tau_i &= \int_{\underline{v}}^{\bar{v}} \int_{\frac{\tau_i}{\tau_j}(\bar{v}-v)+p_j-\bar{v}}^{\bar{\epsilon}} \left[\frac{\tau_j}{\tau_i}(\bar{v} + \epsilon - p_j) \right] d\epsilon dv. \end{aligned}$$

The second-order conditions follow from the baseline model. Simplifying, the equilibrium $(p_1^*, p_2^*, \tau_1^*, \tau_2^*)$ is pinned down by the following system of four equations for $i, j \in \{1, 2\}, i \neq j$.

$$p_i^* = \frac{\bar{\epsilon} + c_i}{2} - \frac{1}{2} \int_{\underline{v}}^{\bar{v}} \left[(\bar{v} - v) \left(\frac{\tau_i^*}{\tau_j^*}(\bar{v} - v) + p_j^* \right) \right] dv + Z \quad (\text{C.1})$$

$$\tau_i^* = \int_{\underline{v}}^{\bar{v}} \int_{\frac{\tau_i^*}{\tau_j^*}(\bar{v}-v)+p_j^*-\bar{v}}^{\bar{\epsilon}} \left[\frac{\tau_j^*}{\tau_i^*}(\bar{v} + \epsilon - p_j^*) \right] d\epsilon dv, \quad (\text{C.2})$$

where $Z \equiv \frac{1}{2} \int_{\underline{v}}^{\bar{v}} [\bar{v}(\bar{v} + \bar{\epsilon}) - v(\bar{v} + \underline{\epsilon})] dv$ is a constant. Brouwer's fixed point argument guarantees the existence of a solution to the system. From (C.1), dividing τ_1^* by τ_2^* and rearranging, we get:

$$\left(\frac{\tau_1^*}{\tau_2^*} \right)^3 = \frac{\int_{\underline{v}}^{\bar{v}} \int_{\frac{\tau_1^*}{\tau_2^*}(\bar{v}-v)+p_2^*-\bar{v}}^{\bar{\epsilon}} [\bar{v} + \epsilon - p_2^*] d\epsilon dv}{\int_{\underline{v}}^{\bar{v}} \int_{\frac{\tau_2^*}{\tau_1^*}(\bar{v}-v)+p_1^*-\bar{v}}^{\bar{\epsilon}} [\bar{v} + \epsilon - p_1^*] d\epsilon dv}. \quad (\text{C.3})$$

From (C.3), it is easy to verify that $\tau_1^* < \tau_2^*$ if and only if $p_1^* < p_2^*$. To prove $\tau_1^* < \tau_2^*$, suppose by contradiction $\tau_1^* \geq \tau_2^*$, which implies $p_1^* \geq p_2^*$. From (C.1), computing the difference $p_1^* - p_2^*$:

$$(p_1^* - p_2^*) \left(1 - \frac{(\bar{v} - \underline{v})^2}{4} \right) = \frac{c_1 - c_2}{2} - \frac{(\bar{v} - \underline{v})^3}{6} \left(\frac{\tau_1^*}{\tau_2^*} - \frac{\tau_2^*}{\tau_1^*} \right) < 0$$

because $c_2 > c_1$ and $\tau_1^* \geq \tau_2^*$. This contradicts (C.3), so we can conclude $\tau_1^* < \tau_2^*$ and $p_1^* < p_2^*$ must hold in equilibrium.

To prove uniqueness, from (C.1) we substitute the expression of p_j^* and rearrange to obtain:

$$\begin{aligned} p_i^* \left(1 - \frac{(\bar{v} - \underline{v})^4}{16} \right) &= \frac{\bar{\epsilon} + c_i}{2} - \frac{1}{2} \int_{\underline{v}}^{\bar{v}} \left[\frac{\tau_i^*}{\tau_j^*}(\bar{v} - v)^2 \right] dv + Z \\ &\quad - \frac{(\bar{v} - \underline{v})^2}{4} \left(\frac{\bar{\epsilon} + c_j}{2} - \frac{1}{2} \int_{\underline{v}}^{\bar{v}} \left[\frac{\tau_j^*}{\tau_i^*}(\bar{v} - v)^2 \right] dv + Z \right), \end{aligned} \quad (\text{C.4})$$

which pins down p_i^* as a decreasing function of $\frac{\tau_i^*}{\tau_j^*}$, implying that p_1^* decreases with $\frac{\tau_1^*}{\tau_2^*}$ while the reverse is true for p_2^* . It follows that the right-hand side of (C.3) is decreasing in $\frac{\tau_1^*}{\tau_2^*}$, hence the solution must be unique. Substituting the unique $\frac{\tau_1^*}{\tau_2^*}$ into (C.4) leads to unique (p_1^*, p_2^*) , which can then be substituted into (C.2) to obtain unique (τ_1^*, τ_2^*) . Finally, we verify (9). The condition clearly holds for $i = 1$ given we have proven $\frac{\tau_2^*}{\tau_1^*} > 1$, and it also holds for $i = 2$ if $\frac{\tau_1^*}{\tau_2^*} < 1$ is close enough to 1, which is true when c_2 is sufficiently small given $\frac{\tau_1^*}{\tau_2^*}$ monotonically increases when c_2 decreases and also $\lim_{c_2 \rightarrow 0} \frac{\tau_1^*}{\tau_2^*} = 1$. ■

In the case of non-uniform distributions we solve numerically for the equilibrium prices, commissions, and condition (9) for each given c_2 . Figure 5 below illustrates the numerical result for F and $G \sim N(\mu, \sigma)$, where $\mu = 0$ and $\sigma = 1$. Consistent with the case of uniform distribution, we observe that $p_1^* < p_2^*$ and $\tau_1^* < \tau_2^*$. In equilibrium (9) holds for all $c_2 \leq 3$, so that consumers indeed find it optimal to follow M 's recommendation, regardless of which firm is recommended. We obtained similar observations for the other values of (μ, σ) that we tried, as well as the exponential distribution.

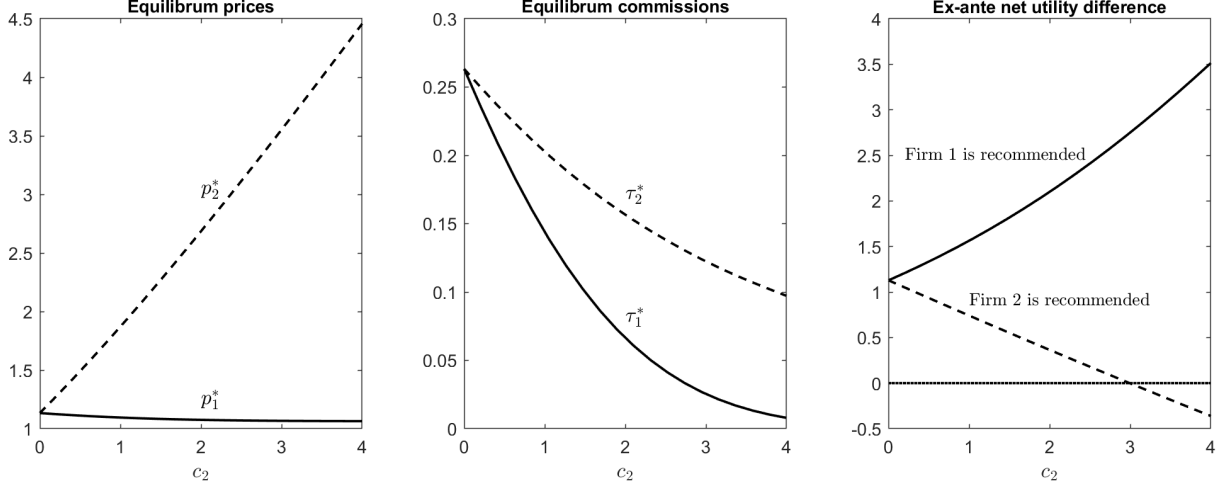


Figure 5: Asymmetric equilibrium, F and $G \sim N(0, 1)$.

C.1.2 The cost of the second search is low

When the cost of the second search is arbitrarily small, we want to prove that there is no pure-strategy equilibrium with asymmetric positive commissions. Consider some arbitrarily given profile of expected and actual prices and commissions, and without loss of generality suppose $0 < \tau_1^* < \tau_2^*$. Similar to the benchmark case, consumers hold passive belief over any unobserved prices and commissions (where applicable). We can construct the following informative equilibrium in the recommendation stage:

- Consumers follow M 's ranking, believing that M ranks all products in order of expected commissions. If firm 1 is ranked first, consumers inspect it without searching further. If firm 2 is ranked first, consumers inspect both products.
- M ranks all products in order of expected commission $\tau_i(1 - G(p_i - \epsilon_i))$.

Clearly, consumers' search strategy is optimal given their beliefs and M 's equilibrium strategy. To verify M 's strategy, note if $\epsilon_1 - p_1 \leq \epsilon_2 - p_2$ then the profit from ranking firm 1 first is $\tau_1(1 - G(p_1 - \epsilon_1))$ and from ranking firm 2 first is $\tau_2(1 - G(p_2 - \epsilon_2))$. If instead $\epsilon_1 - p_1 > \epsilon_2 - p_2$ then M 's profit is always $\tau_1(1 - G(p_1 - \epsilon_1))$ regardless of the recommendation because consumers always inspect both products and buy from firm 1 whenever firm 2 is ranked first. In this case, the equilibrium strategy specifies that M breaks a tie in favor of the firm with higher $\tau_i(1 - G(p_i - \epsilon_i))$, which is required for the equilibrium to exist. From the informative equilibrium in the recommendation stage, firm 2's demand is

$$\Pr\left(\epsilon_2 - p_2 > \max\left\{\epsilon_1 - p_1, -G^{-1}\left(1 - \frac{\tau_1}{\tau_2}(1 - G(p_1 - \epsilon_1))\right), -v\right\}\right).$$

Notice that for all $\tau_2 > \tau_1$, any increase in τ_2 has no effect on the demand because $-G^{-1}\left(1 - \frac{\tau_1}{\tau_2}(1 - G(p_1 - \epsilon_1))\right) < \epsilon_1 - p_1$. Therefore, firm 2 never sets $\tau_2 > \tau_1$ as opposed to the initial supposition, i.e. any equilibrium with $0 < \tau_1^* < \tau_2^*$ is not sustainable. A mirror argument shows that equilibrium with $\tau_1^* > \tau_2^* > 0$ is also not sustainable, as claimed in the text.

C.1.3 Two groups of consumers

Suppose there are two groups of consumers: a fraction λ of which have arbitrarily small cost for the second search (*low-cost consumers*) while the remaining fraction $1 - \lambda$ have sufficiently high cost for the second search and they only search once (*high-cost consumers*). We now construct the asymmetric (pure-strategy) informative equilibrium with steering.

Proposition 15 *Suppose F and G are linear. For each given $c_2 - c_1 > 0$ such that Proposition 7 holds, if λ is sufficiently small then there exists an asymmetric informative equilibrium with steering in which:*

1. *Firms set prices $p_1^* < p_2^*$ and commissions $\tau_1^* < \tau_2^*$.*
2. *M ranks all products in order of expected commission $\tau_i (1 - G(p_i - \epsilon_i))$.*
3. *All high-cost consumers inspect the recommended product without searching further.*
4. *Low-cost consumers inspect the highest ranked product first. If firm 1 is ranked first, they inspect it without searching further. If firm 2 is ranked first, they inspect both products. They believe that M ranks all products in order of expected commission.*
5. *All consumers believe that M ranks all products in order of expected commission. In case M makes no recommendation, consumers' purchase and search behavior is optimized as if M is absent.*

Proof. We first check M 's incentives regarding its ranking. If $\epsilon_1 - p_1 \leq \epsilon_2 - p_2$ then the profit from ranking firm 1 first is $\tau_1 (1 - G(p_1 - \epsilon_1))$ and from ranking firm 2 first is $\tau_2 (1 - G(p_2 - \epsilon_2))$. If instead $\epsilon_1 - p_1 > \epsilon_2 - p_2$, then M 's profit from ranking firm 1 first is $\tau_1 (1 - G(p_1 - \epsilon_1))$ and from ranking firm 2 first is $\lambda \tau_1 (1 - G(p_1 - \epsilon_1)) + (1 - \lambda) \tau_2 (1 - G(p_2 - \epsilon_2))$. In both cases, M does best by ranking all products in order of expected commission $\tau_i (1 - G(p_i - \epsilon_i))$. Then, it remains to check consumers indeed find it optimal to follow M 's ranking, i.e. check whether (9) indeed holds in equilibrium. From the informative equilibrium in the recommendation stage, we can derive firm 1's demand, which is the same as in the baseline model because all consumers only inspect once whenever firm 1 is ranked first. Firm 2's demand becomes

$$D_2(p_2, \tau_2) = \left(\begin{array}{l} (1 - \lambda) \Pr \left(\epsilon_2 - p_2 > \max \left\{ -G^{-1} \left(1 - \frac{\tau_1}{\tau_2} (1 - G(p_1 - \epsilon_1)) \right), -v \right\} \right) \\ + \lambda \Pr \left(\epsilon_2 - p_2 > \max \left\{ \epsilon_1 - p_1, -G^{-1} \left(1 - \frac{\tau_1}{\tau_2} (1 - G(p_1 - \epsilon_1)) \right), -v \right\} \right) \end{array} \right).$$

Specifically, there is an extra term $\epsilon_1 - p_1$ in the second demand component because low-cost consumers inspect both products. In general, $D_2(p_2, \tau_2)$ is continuous in τ_2 but not differentiable at $\tau_2 = \tau_1$ because $\frac{dD_2}{d\tau_2} \Big|_{\tau_2 \rightarrow \tau_1^-} = \frac{1}{(1-\lambda)} \frac{dD_2}{d\tau_2} \Big|_{\tau_2 \rightarrow \tau_1^+}$, i.e. the slope of the demand function has a downward kink at $\tau_2 = \tau_1$. Nonetheless, given that we are focusing on an equilibrium with $\tau_1^* < \tau_2^*$, firm 2's commission is necessarily an interior one and pinned down by $-\frac{dD_2}{dp_2} = \frac{dD_2}{d\tau_2} \Big|_{\tau_2 > \tau_1}$, as otherwise the equilibrium is violated. Therefore, we write firm 2's profit function as

$$\Pi_2 = (p_2 - c_2 - \tau_2) \left[\begin{array}{l} (1 - \lambda) \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[1 - \frac{(\max\{\bar{x}_2(\epsilon), -v\}) + p_2 - \epsilon}{\bar{\epsilon} - \underline{\epsilon}} \right] \left(\frac{1}{\bar{\epsilon} - \underline{\epsilon}} \right) \left(\frac{1}{\bar{v} - \underline{v}} \right) d\epsilon dv \\ + \lambda \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[1 - \frac{(\max\{\epsilon_1 - p_1, -v\}) + p_2 - \epsilon}{\bar{\epsilon} - \underline{\epsilon}} \right] \left(\frac{1}{\bar{\epsilon} - \underline{\epsilon}} \right) \left(\frac{1}{\bar{v} - \underline{v}} \right) d\epsilon dv \end{array} \right].$$

Then, the first-order conditions for firm 2 can be derived as:

$$\begin{aligned} p_2^* &= c_2 + \tau_2^* + (1 - \lambda) \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\bar{\epsilon} - \left(\max \left\{ \frac{\tau_1^*}{\tau_2^*} (\bar{v} + \epsilon - p_1^*) - \bar{v}, -v \right\} \right) - p_2^* \right] d\epsilon dv \\ &\quad + \lambda \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} [\bar{\epsilon} - (\max\{\epsilon - p_1^*, -v\}) - p_2^*] d\epsilon dv \\ \tau_2^* &= (1 - \lambda) \int_{\underline{v}}^{\bar{v}} \int_{\frac{\tau_1^*}{\tau_2^*}(\bar{v}-v)+p_1^*-\bar{v}}^{\bar{\epsilon}} \left[\frac{\tau_1^*}{\tau_2^*} (\bar{v} + \epsilon - p_1^*) \right] d\epsilon dv, \end{aligned}$$

while the first-order conditions for firm 1 are (C.1) and (C.2) (setting $i = 1$). Given firm 2's best responses are continuous in λ and converges to (C.1) and (C.2) (setting $i = 2$), it follows from continuity that for sufficiently small λ the equilibrium in Proposition 7 holds. In particular, the equilibrium $(p_1^*, p_2^*, \tau_1^*, \tau_2^*)$ is such that condition (9) holds for $i = 1, 2$ and also $\tau_1^* < \tau_2^*$, as required. ■

For an illustration, the figure below plots the equilibrium outcome as a function of λ , assuming F and $G \sim U[-1, 1]$, and $c_2 = 0.5$. We observe $\tau_1^* < \tau_2^*$ for all $\lambda \leq 0.35$ such that the asymmetric informative equilibrium exists. For $\lambda > 0.35$, $\tau_1^* = \tau_2^*$ implies that the asymmetric informative equilibrium no longer exists. A similar observation can be obtained assuming F and $G \sim N(0, 1)$, and $c_2 = 1$, in which case $\tau_1^* < \tau_2^*$ for all $\lambda \leq 0.5$ such that the asymmetric informative equilibrium exists.

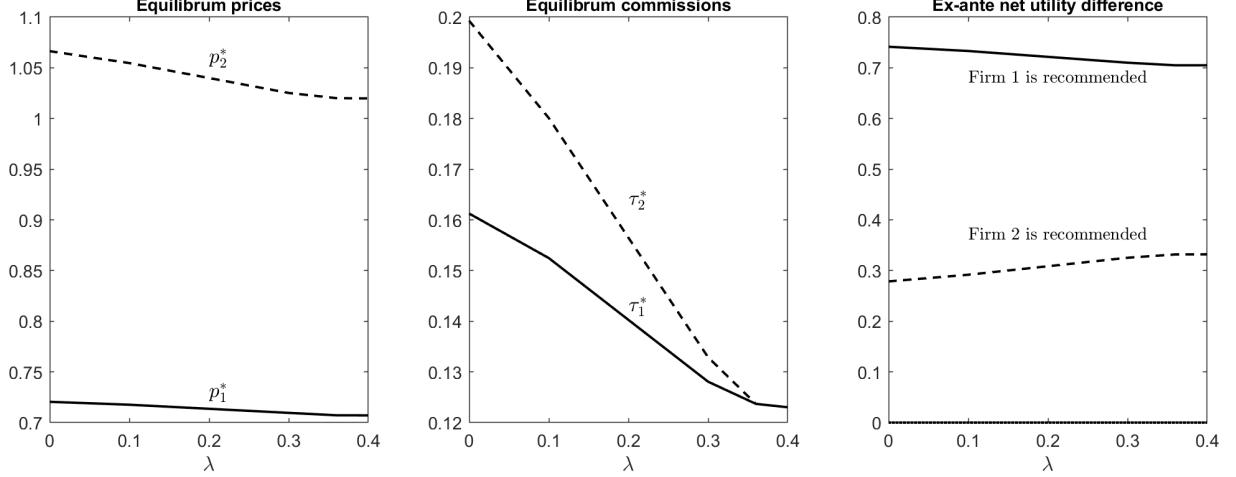


Figure 6: Asymmetric equilibrium, F and $G \sim U[-1, 1]$ and $c_2 = 0.5$.

C.2 Fee-setting intermediary

For any given τ set by M , the pricing stage among n firms is simply the Perloff-Salop model where a typical firm i solves

$$\max_{p_i} (p_i - \tau) \int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} [1 - F(\max\{\epsilon, -v\} + p_i)] dF(\epsilon)^{n-1} dG(v).$$

From the first-order conditions, the equilibrium price $p^* = p^*(\tau)$ satisfies

$$p^* = \tau + \underbrace{\frac{\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} [1 - F(\max\{\epsilon, p^* - v\})] dF(\epsilon)^{n-1} dG(v)}{\int_{\underline{v}}^{\bar{v}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} f(\max\{\epsilon, p^* - v\}) dF(\epsilon)^{n-1} dG(v)}}_{\equiv \phi_1(p^*)}. \quad (\text{C.5})$$

We note that firms' second-order conditions hold when f and g are log-concave.

Next, consider M 's fee-setting problem. Since M collects a fee for all transactions, the total demand it faces is the sum of demands of all n firms, or simply the total market coverage of all n firms. Hence, it sets a commission that solves

$$\max_{\tau} \left\{ \tau \int_{\underline{\epsilon}}^{\bar{\epsilon}} [1 - G(p^* - \epsilon)] dF^n(\epsilon) \right\} \quad (\text{C.6})$$

subject to $p^* = \tau + \phi_1(p^*)$.

We are now ready to prove Proposition 8 in the main text.

Proof. (Proposition 8). We first consider the limiting result. Based on the pricing constraint, we can recast M 's maximization problem as choosing final product prices directly. This is possible because $\phi_1(\cdot)$ is a strictly decreasing function (by the last part of the proof of Proposition 1), hence there is a one-to-one

relationship between p^* and τ with $dp^*/d\tau \in [-1, 0]$. Hence, M solves

$$\max_p (p - \phi_1(p)) \int_{\underline{\epsilon}}^{\bar{\epsilon}} [1 - G(p - \epsilon)] dF^n(\epsilon). \quad (\text{C.7})$$

Equivalently, M is a monopolist who sells a product with valuation $\max_{j=1, \dots, n} \{\epsilon_j\}$ and faces a marginal cost at $\phi_1(p)$. When n approaches infinity, $\phi_1(p) \rightarrow 0$ while at the same time the distribution F collapses to a single point at $\bar{\epsilon}$ so that (C.7) becomes

$$\max_p p(1 - G(p - \bar{\epsilon})).$$

We have $\tau^p \equiv \arg \max_p p(1 - G(p - \bar{\epsilon})) = \tau^m$ by the definition of τ^m .

In what follows, define $p^*(\tau)$ as the solution to $p^* = \tau + \phi_1(p^*)$ for each given τ . When both F and G are linear, the associated first-order condition for (C.6) that pins down τ^p is

$$\begin{aligned} \tau &= \frac{\int_{\underline{\epsilon}}^{\bar{\epsilon}} [1 - G(p^*(\tau) - \epsilon)] dF^n(\epsilon)}{-\frac{dp^*}{d\tau} \int_{\underline{\epsilon}}^{\bar{\epsilon}} [g(p^*(\tau) - \epsilon)] dF^n(\epsilon)} \\ &= \frac{-1}{dp^*/d\tau} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\frac{1 - G(p^*(\tau) - \epsilon)}{g(p^*(\tau) - \epsilon)} \right] \left[\frac{g(p^*(\tau) - \epsilon)}{\int_{\underline{\epsilon}}^{\bar{\epsilon}} g(p^*(\tau) - \epsilon) dF^n(\epsilon)} \right] dF^n(\epsilon) \\ &= \frac{-1}{dp^*/d\tau} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\frac{1 - G(p^*(\tau) - \epsilon)}{g(p^*(\tau) - \epsilon)} \right] \left[\frac{1}{1 - F(p^*(\tau))^n} \right] dF^n(\epsilon), \end{aligned}$$

where the last equality utilizes that g is constant when G is linear. Meanwhile, recall that from (7) that after substituting for constant $f(\epsilon) = g(\epsilon)$ and changing the order of integration, the equilibrium commission τ^* (when firms set commission) is pinned down by

$$\tau = \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\frac{1 - G(p^*(\tau) - \epsilon)}{g(p^*(\tau) - \epsilon)} \right] \left[\frac{1 - G(p^*(\tau) - \epsilon)}{1 - G(p^*(\tau) - \bar{\epsilon})} \right] dF^{n-1}(\epsilon). \quad (\text{C.8})$$

To show $\tau^p \geq \tau^*$, we note that for any $\tau \geq 0$,

$$\begin{aligned} &\frac{-1}{dp^*/d\tau} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\frac{1 - G(p^*(\tau) - \epsilon)}{g(p^*(\tau) - \epsilon)} \right] \left[\frac{1}{1 - F(p^*(\tau))^n} \right] dF^n(\epsilon) \\ &\geq \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\frac{1 - G(p^*(\tau) - \epsilon)}{g(p^*(\tau) - \epsilon)} \right] dF^n(\epsilon) \\ &\geq \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\frac{1 - G(p^*(\tau) - \epsilon)}{g(p^*(\tau) - \epsilon)} \right] dF^{n-1}(\epsilon) \\ &\geq \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\frac{1 - G(p^*(\tau) - \epsilon)}{g(p^*(\tau) - \epsilon)} \right] \left[\frac{1 - G(p^*(\tau) - \epsilon)}{1 - G(p^*(\tau) - \bar{\epsilon})} \right] dF^{n-1}(\epsilon), \end{aligned}$$

where the first inequality is due to $\frac{-1}{dp^*/d\tau} \left(\frac{1}{1 - F(p^*(\tau))^n} \right) \geq 1$ (because $dp^*/d\tau \in [-1, 0]$), the second inequality is due to first-order stochastic dominance, while the third inequality is due to $\left[\frac{1 - G(p^*(\tau) - \epsilon)}{1 - G(p^*(\tau) - \bar{\epsilon})} \right] \leq 1$. The final line of the expression is exactly the RHS (C.8), and it is decreasing in τ . We thus conclude that $\tau^p \geq \tau^*$. ■